

Stochastic Problems in Dynamics. B. L. Clarkson (Editor). Pitman, London, San Francisco, Melbourne. 1977. Pages 566. Price \$22.50.

REVIEWED BY T. K. CAUGHEY¹

This volume constitutes the *Proceedings of the I. U. T. A. M. Symposium on Stochastic Problems in Dynamics*, held at the University of Southampton, England, in July 1976. There are a total of 30 papers on such diverse topics as stochastic stability, first-passage probability, flutter and random vibration of plates, and nonlinear response of ships on the sea.

Kistner considers the parametric excitation of linear systems by stationary Gaussian colored noise processes, and derives closed-form expressions for the first and second moments of the state variable, these are used to show the evolution of the moments for finite time. Moment stability is discussed in the paper by Willems who gives criteria for almost-sure stability.

In the field of control theory, Nakamizo and Oshiro consider an optimal stochastic control problem for a linear dynamical system with quadratic performance index, for which the matrix weighting the control and observation noise intensity matrix are singular.

Ariaratnam and Tam use the method of stochastic averaging, due to Stratonovich and Khaminskii, to investigate the stability of coupled linear systems under combined harmonic and stochastic parametric excitation.

The first-passage problem is considered in papers by Grossmeyer and Roberts. Grossmeyer obtains upper and lower bounds for the first-passage probability density and applies his results to the problem of calculating the reliability of structures under earthquake excitation. Roberts predicts the first-passage probabilities for lightly damped oscillators (with linear and nonlinear damping and restoring forces) by considering the energy envelope of the process, which is modeled as a one-dimensional Markov process.

Crandall shows that when a uniform structure such as a plate of rectangular, circular, or triangular form is excited by broad band noise, the response shows surprisingly regular features, which disappear when the plate is of irregular shape.

There are three papers on ship dynamics, including the paper by Yamanouchi, who discusses the nonlinear response of ships on a random sea.

In a very interesting paper Kozin and Sugimoto consider the problem of developing criteria for determining the stability of a stochastic system from observed data. The authors present the results of some computer simulations which clearly demonstrate the difficulty of determining by observation when a stochastic system is stable or unstable.

Each paper is followed by a discussion, and in a number of cases, the discussions highlighted controversies and unexplained effects which should form the basis for future research.

Asymptotic Analysis of Periodic Structures. By Alain Bensoussan, Jacques-Louis Lions, and George Papanicolaou. 1978. North Holland Publishing Company, Amsterdam/New York. Pages 700. Price \$49.

REVIEWED BY T. K. CAUGHEY²

Many problems in engineering and science, such as the study of

composite materials, macroscopic properties of crystalline or polymer structures, nuclear reactor core design, etc., lead to the study of boundary-value problems in media with periodic, or almost periodic, structure. If the period of the microstructure is small compared to the spatial extent of the structure, one often tries to derive a macroscopic description of the behavior of the system, given the microscopic description of the structure.

This book considers four methods for solving such problems. The first method is based on the construction of asymptotic expansions using multiple scales. In the present context there are at least two natural spatial length scales, the fast scale measuring variations within the microstructure, and the slow scale measuring variations within the macrostructure. The second method is based on energy estimates and is often used in conjunction with the first method. The third method is based on probabilistic arguments and works whenever the problem admits a probabilistic formulation or has a probabilistic origin. The fourth method is based on the spectral decomposition of operators with periodic coefficients and involves expansions in Bloch waves. This technique is used in the study of high frequency wave propagation in rapidly varying periodic media.

This book is organized into three sections which can be read independently by readers with specialized interests. Chapters one and two form the first section which deals with elliptic, parabolic, and hyperbolic (but not high frequency) problems with emphasis on methods one and two. Parts one, two, and three of Chapter one are basic to the whole book. Chapter three forms the second section and treats problems probabilistically using method three. A separate introduction to the contents of this section is given at the beginning of this chapter. Chapter four forms the third section and deals with high frequency problems using method four. A separate introduction is also provided for this chapter.

This book is written for readers with a good mathematical preparation and may not appeal to some engineers and scientists whose interest in the subject is more applied than theoretical.

Finite-Element Method: Basic Technique and Implementation.

By Pin Tong and John N. Rossettos. 1977. The MIT Press. Pages x-332. Price \$19.95.

REVIEWED BY I. FRIED³

The authors intend this book to serve in an introductory course on the technical aspects of the finite-element method given to engineering students; and it is so. Whether the book will appeal also to the more general audience of applied mathematicians, as the authors hope it will, or even to the more theoretically inclined, talented and curious engineering student, is doubtful.

Nevertheless, the book is readable and logically structured. Chapter 1 mentions the variational origin of finite elements, presents the principal aspects of the technique on a one-dimensional problem, and fixes the basic terminology. Chapter 2 extends the discussion of Chapter 1 to the two-dimensional membrane problem discretized with triangular, first-order elements. Assembly of the global stiffness matrix is detailed, as well as the introduction of the essential (rigid the authors call them) and natural boundary conditions. Further

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Thin periodic structures depend on two interrelated small geometric parameters $\hat{\mu}$ and $h(\hat{\mu})$ which control the thickness of constituents and the cell of periodicity. We study homogenisation of elasticity theory problems on these structures by method of asymptotic expansions. A particular attention is paid to the case of critical thickness when $\lim_{\hat{\mu} \rightarrow 0} h(\hat{\mu})\hat{\mu}^{-1}$ is a positive constant. We determine the asymptotic behaviour of the electromagnetic response of such a medium in the "homogenisation limit", as $\hat{\mu} \rightarrow 0$; and derive the limit system of Maxwell equations in \mathbb{R}^3 : Our results extend a number of conclusions of the paper [Zhikov, V. V., 2004. On the band-gap structure of the spectrum of some divergent-form elliptic operators with periodic coefficients, St. Petersburg Math.