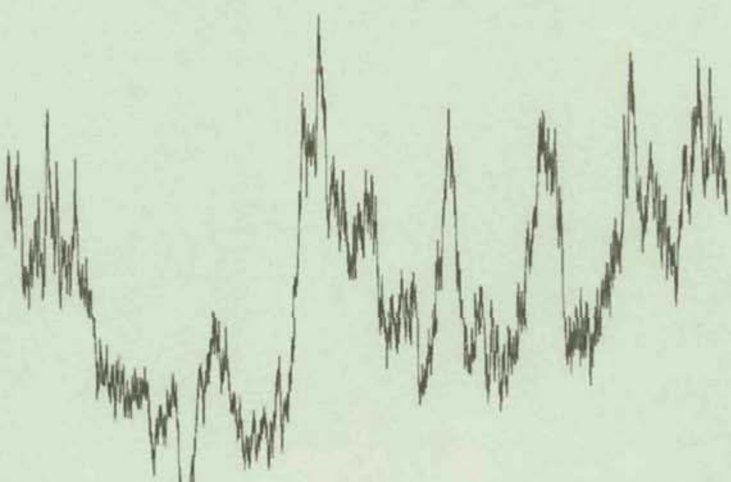
Advanced Series on Statistical Science & Applied Probability

Vol. 2

Ruin Probabilities

Søren Asmussen



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Ruin Probabilities

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Contents

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Preface

The most important to say about the history of this book is: it took too long time to write it! In 1991, I was invited to give a course on ruin probabilities at the Laboratory of Insurance Mathematics, University of Copenhagen. Since I was to produce some hand–outs for the students anyway, the idea was close to expand these to a short book on the subject, and my belief was that this could be done rather quickly.

The course was never realized, but the hand–outs were written and the book was started (even a contract was signed with a deadline I do not dare to write here!). But the pace was much slower than expected, and other projects absorbed my interest. As an excuse: many of these projects were related to the book, and the result is now that the book is much more related to my own research than the initial outline.

Let me take this opportunity to thank above all my publisher World Scientific Publishing Co. and the series editor Ole Barndorff–Nielsen for their patience. A similar thank goes to all colleagues who encouraged me to finish the project and continued to refer to the book by Asmussen which was to appear in a year which continued to be postponed.

Risk theory in general and ruin probablities in particular is traditionally considered as part of insurance mathematics, and has been an active area of research from the days of Lundberg all the way up to today. However, it would not be fair not to say that the practical relevance of the area has been questioned repeatedly. One reason for writing this book is a feeling that the area has in the recent years achieved a considerable mathematical maturity, which has in particular removed one of the standard criticisms of the area, that it can only say something about very simple models and questions. Apart from these remarks, I have deliberately stayed away from discussing the practical relevance of the theory; if the formulations occasionally give a different impression, it is not by intention. Thus, the book is basically mathematical in its flavour.

It has obviously not been possible to cover all subareas. In particular, this applies to long–range dependence which is intensely studied in the neighboring field of queueing theory. The main motivation comes from statistical data for network traffic (e.g. Willinger *et al.* [381]); for the effects on tail probabilities, see e.g. Resnick & Samorodnitsky [303] and references therein. Concerning ruin probabilities, see in particular Michna [259]. Another interesting area which is not covered is dynamic control. In the classical setting of Cramér–Lundberg models, some basic discussion can be found in the books by Bühlmann [82] and Gerber [157]; see also Schmidli [325] and the references in Asmussen & Taksar [52]. More recently, the standard stochastic control setting of diffusion models has been considered, e.g. Højgaard & Taksar [206], Asmussen, Højgaard & Taksar [35] and Paulsen & Gjessing [284]. The book does not go into the broader aspects of the interface between insurance mathematics and mathematical finance, an area which is becoming increasingly important. Finally, I regret that due to time constraints, it has not been possible to incorporate more numerical examples than the few there are.

A book like this can be organized in many ways. One is by model, another by method. The present book is in between these two possibilities. Chapters III–VII introduce some of the main models and give a first derivation of some of their properties. Chapters IX–X then go in more depth with some of the special approaches for analyzing specific models and add a number of results on the models in Chapters III–VII (also Chapter II is essentially methodological in its flavor).

Here is a suggestion on how to get started with the book. For a brief orientation, read Chapter I, the first part of II.6 (to understand the Pollaczeck-Khinchine formula in III.2 more properly), III.1–5, IV.4a, VII.1, VIII.1–3 and IX.1–3. For a second reading, incorporate II.1–4, III.8–9, IV.2, IV.5, VI.1–3, VII.2, IX.4–5, X.1–3 and XI.3. The rest is up to your specific interests. Good luck!

I have tried to be fairly exhaustive in citing references close to the text, In addition, some papers not cited in the text but judged to be of interest are included in the Bibliography. It is obvious that such a system involves a number of inconsistencies and omissions, for which I apologize to the reader and the authors of the many papers who ought to have been on the list.

I intend to keep a list of misprints and remarks posted on my web page,

`http://www.maths.lth.se/matstat/staff/asmus`

and I am therefore grateful to get relevant material sent by email to

`asmus@maths.lth.se`

Lund February 2000
Søren Asmussen

The second printing differs from the first only by minor corrections, many of which were pointed out by Hanspeter Schmidli. More substantial remarks, of which there are not many at this stage, as well as some additional references continue to be at the web page.

Lund September 2001
Søren Asmussen

Acknowledgements

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Section VII.3 is reprinted from Asmussen & Nielsen [39] and parts of IX.4 from Asmussen, Schmidli & Schmidt [47] with the permission from Applied Probability Trust. Section VIII.1 is almost identical to Section 2 of Asmussen [26] and reprinted with permission of Blackwell Publishers. Parts of II.6 is reprinted from Asmussen & Schmidt [49] and parts of IX.5 from Asmussen & Klüppelberg [36] with the permission from Elsevier Science. Parts of X.1 and X.3 are reprinted from Asmussen & Rubinstein [46] and parts of VIII.5 from Asmussen [21] with permission from CRC Press.

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Chapter I

Introduction

1 The risk process

In this chapter, we introduce some general notation and terminology, and give a very brief summary of some of the models, results and topics to be studied in the rest of the book.

A *risk reserve process* $\{R_t\}_{t \geq 0}$, as defined in broad terms, is a model for the time evolution of the reserves of an insurance company. We denote throughout the initial reserve by $u = R_0$. The probability $\psi(u)$ of ultimate ruin is the probability that the reserve ever drops below zero,

$$\psi(u) \;=\; \mathbf{P}\left(\inf_{t \geq 0} R_t < 0\right) \;=\; \mathbf{P}\left(\inf_{t \geq 0} R_t < 0 \,\middle|\, R_0 = u\right). \tag{1.1}$$

The probability of ruin before time T is

$$\psi(u, T) \;=\; \mathbf{P}\left(\inf_{0 \leq t \leq T} R_t < 0\right). \tag{1.2}$$

We also refer to $\psi(u)$ and $\psi(u, T)$ as ruin probabilities with infinite horizon and finite horizon, respectively. *They are the main topics of study of the present book.*

For mathematical purposes, it is frequently more convenient to work with the *claim surplus process* $\{S_t\}_{t \geq 0}$ defined by $S_t = u - R_t$. Letting

$$\tau(u) \;=\; \inf\{t \geq 0 : R_t < 0\} \;=\; \inf\{t \geq 0 : S_t > u\}, \tag{1.3}$$

$$M \;=\; \sup_{0 \leq t < \infty} S_t, \qquad M_T \;=\; \sup_{0 \leq t \leq T} S_t, \tag{1.4}$$

be the time to ruin and the maxima with infinite and finite horizon, respectively, the ruin probabilities can then alternatively be written as

$$\psi(u) = \mathbf{P}(\tau(u) < \infty) = \mathbf{P}(M > u), \tag{1.5}$$

$$\psi(u,T) = \mathbf{P}(M_T > u) = \mathbf{P}(\tau(u) \le T). \tag{1.6}$$

Sofar we have not imposed any assumptions on the risk reserve process. However, the following set-up will cover the vast majority of the book:

- There are only finitely many claims in finite time intervals. That is, the number N_t of arrivals in $[0,t]$ is finite. We denote the interarrival times of claims by $T_2, T_3, \ldots$ and T_1 is the time of the first claim. Thus, the time of arrival of the nth claim is $\sigma_n = T_1 + \cdots + T_n$, and $N_t = \min\{n \ge 0 : \sigma_{n+1} > t\} = \max\{n \ge 0 : \sigma_n \le t\}$.
- The size of the nth claim is denoted by U_n.
- Premiums flow in at rate p, say, per unit time.

Putting things together, we see that

$$R_t = u + pt - \sum_{k=1}^{N_t} U_k, \quad S_t = \sum_{k=1}^{N_t} U_k - pt. \tag{1.7}$$

The sample paths of $\{R_t\}$ and $\{S_t\}$ and the connection between the two processes are illustrated in Fig. 1.1.

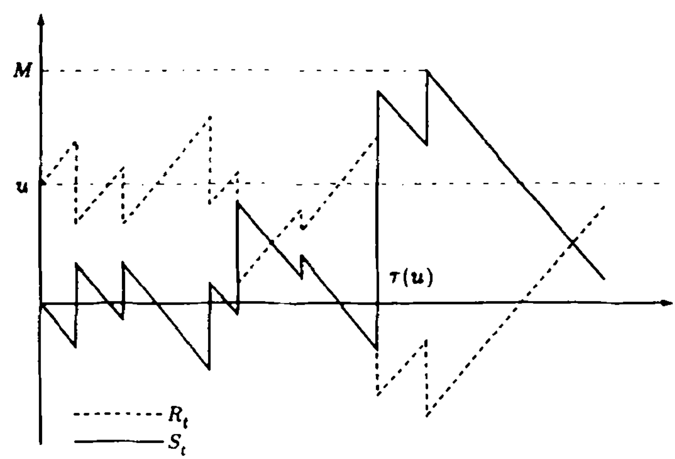

Figure 1.1

Yuliya Mishura, Olena Ragulina. Ruin Probabilities: Smoothness, Bounds, Supermartingale Approach deals with continuous-time risk models and covers several aspects of risk theory. The first of them is the smoothness of the survival probabilities. In particular, the book provides a detailed investigation of the continuity and differentiability of the infinite-horizon and finite-horizon survival probabilities for different risk models. Some important results about ruin probabilities are obtained by martingale approach. 1. Introduction. Mixture models are a fundamental tool in applied statistics, for most mixture models, including the widely used mixtures of Gaussians and hidden Markov models (HMMs); the current practice relies on the Expectation-Maximization (EM) algorithm, a local search heuristic for maximum likelihood estimation; an efficient method of moments approach to parameter estimation for a broad class of high-dimensional mixture models.