

Hermann Weyl

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Hermann Klaus Hugo Weyl FRS^[2] (9 November 1885 – 8 December 1955) was a German mathematician and theoretical physicist. Although much of his working life was spent **in Zürich**, Switzerland and **then Princeton**, he is associated with the University of Göttingen tradition of mathematics, represented by David Hilbert and Hermann Minkowski. His research has had major significance for theoretical physics as well as purely mathematical disciplines including number theory. He was one of the most influential mathematicians of the twentieth century, and an important member of the Institute for Advanced Study during its early years.^{[3][4][5]}

Weyl published technical and some general works on space, time, matter, philosophy, logic, symmetry and the history of mathematics. He was one of the first to conceive of combining general relativity with the laws of electromagnetism. While **no** mathematician of his generation aspired to the **'universalism'** of **Henri Poincaré** or **Hilbert**, Weyl came as close as anyone. Michael Atiyah, in particular, has commented that whenever he examined a mathematical topic, he found that Weyl had preceded him (*The Mathematical Intelligencer* (1984), vol.6 no.1).

Hermann Klaus Hugo Weyl



Born	9 November 1885 Elmshorn, Germany
Died	8 December 1955 (aged 70) Zurich, Switzerland
Fields	Mathematical physics
Institutions	Institute for Advanced Study University of Göttingen ETH Zurich
Alma mater	University of Göttingen
Doctoral advisor	David Hilbert ^[1]
Notable students	Saunders Mac Lane
Known for	See list of topics named after Hermann Weyl
Notable awards	Fellow of the Royal Society ^[2]

Signature

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Biography

Weyl was born in Elmshorn, a small town near Hamburg, in Germany, and attended the *gymnasium* Christianeum in Altona.^[6]

From 1904 to 1908 he studied mathematics and physics in both Göttingen and Munich. His doctorate was awarded at the University of Göttingen under the supervision of David Hilbert whom he greatly admired. After taking a teaching post for a few years, he left Göttingen for Zürich to take the chair of mathematics in the ETH Zurich, where he was a colleague of Albert Einstein, who was working out the details of the theory of general relativity. Einstein had a lasting influence on Weyl who became fascinated by mathematical physics. Weyl met Erwin Schrödinger in 1921, who was appointed Professor at the University of Zürich. They were to become close friends over time.

Weyl left Zürich in 1930 to become Hilbert's successor at Göttingen, leaving when the Nazis assumed power in 1933, particularly as his wife was Jewish. The events persuaded him to move to the new Institute for Advanced Study in Princeton, New Jersey. He remained there

until his retirement in 1951. Together with his wife, he spent his time in Princeton and Zürich, and died in Zürich in 1955.

Contributions

Distribution of eigenvalues

Further information: Weyl law, Weyl conjecture, Asymptotic distribution of eigenvalues

In 1911 Weyl published *Über die asymptotische Verteilung der Eigenwerte* (*On the asymptotic distribution of eigenvalues*) in which he proved that the eigenvalues of the Laplacian in the compact domain are distributed according to Weyl law. In 1912 he suggested a new proof, based on variational principles. Weyl returned to this topic several times, considered elasticity system and formulated Weyl conjecture These works started an important domain *Asymptotic distribution of eigenvalues* of Modern Analysis.

Geometric foundations of manifolds and physics

Further information: Weyl transformation, Weyl tensor

In 1913, Weyl published *Die Idee der Riemannschen Fläche* (*The Concept of a Riemann Surface*), which gave a unified treatment of Riemann surfaces. In it Weyl utilized point set topology, in order to make Riemann surface theory more rigorous, a model followed in later work on manifolds. He absorbed L. E. J. Brouwer's early work in topology for this purpose.

Weyl, as a major figure in the Göttingen school, was fully apprised of Einstein's work from its early days. He tracked the development of relativity physics in his *Raum, Zeit, Materie* (*Space, Time, Matter*) from 1918, reaching a 4th edition in 1922. In 1918, he introduced the notion of gauge, and gave the first example of what is now known as a gauge theory. Weyl's gauge theory was an **unsuccessful** attempt to model **the electromagnetic field** and the gravitational field as geometrical properties of spacetime. The Weyl tensor in Riemannian geometry is of major importance in understanding the nature of conformal geometry. In 1929, Weyl introduced the concept of the vierbein into general relativity.^[7]



Hermann Weyl (left) and Ernst Peschl (right).

His overall approach in physics was based on the phenomenological philosophy of Edmund Husserl, specifically Husserl's 1913 *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie. Erstes Buch: Allgemeine Einführung in die reine Phänomenologie* (Ideas of a Pure Phenomenology and Phenomenological Philosophy. First Book: General Introduction). Apparently this was Weyl's way of dealing with Einstein's controversial dependence on the phenomenological physics of Ernst Mach.^[*citation needed*]

Husserl had reacted strongly to Gottlob Frege's criticism of his first work on the philosophy of arithmetic and was investigating the sense of mathematical and other structures, which Frege had distinguished from empirical reference. Hence there is good reason for viewing gauge theory as it developed from Weyl's ideas as a formalism of physical measurement and not a theory of anything physical, i.e. as scientific formalism.^[*citation needed*]

Topological groups, Lie groups and representation theory

Main articles: Peter–Weyl theorem, Weyl group, Weyl spinor, and Weyl algebra

From 1923 to 1938, Weyl developed the theory of compact groups, in terms of matrix representations. In the compact Lie group case he proved a fundamental character formula.

These results are foundational in understanding the symmetry structure of quantum mechanics, which he put on a group-theoretic basis. This included spinors. Together with the mathematical formulation of quantum mechanics, in large measure due to John von Neumann, this gave the treatment familiar since about 1930. Non-compact groups and their representations, particularly the Heisenberg group, were also streamlined in that specific context, in his 1927 Weyl quantization, the best extant bridge between classical and quantum physics to date. From this time, and certainly much helped by Weyl's expositions, Lie groups and Lie algebras became a mainstream part both of pure mathematics and theoretical physics.

His book *The Classical Groups*, a seminal if difficult text, reconsidered invariant theory. It covered symmetric groups, general linear groups, orthogonal groups, and symplectic groups and results on their invariants and representations.

Harmonic analysis and analytic number theory

For more details on this topic, see Weyl's criterion.

Weyl also showed how to use exponential sums in diophantine approximation, with his criterion for uniform distribution mod 1, which was a fundamental step in analytic number theory. This work applied to the Riemann zeta function, as well as additive number theory. It was developed by many others.

Foundations of mathematics

In *The Continuum* Weyl developed the logic of predicative analysis using the lower levels of Bertrand Russell's ramified theory of types. He was able to develop **most** of **classical calculus**, while using **neither** the axiom of choice **nor** proof by contradiction, and **avoiding** Georg Cantor's infinite sets. Weyl appealed in this period to the radical constructivism of the German romantic, subjective idealist Fichte.

Shortly after publishing *The Continuum* Weyl briefly shifted his position wholly to the **intuitionism of Brouwer**. In *The Continuum*, the constructible points exist as discrete entities. Weyl wanted a continuum that was not an aggregate of points. He wrote a controversial article proclaiming that, for himself and L. E. J. Brouwer, "We are the revolution." This article was far more influential in propagating **intuitionistic** views than the original works of Brouwer himself.

George Pólya and Weyl, during a mathematicians' gathering in Zürich (9 February 1918), made a bet concerning the future direction of mathematics. Weyl predicted that in the subsequent 20 years, mathematicians would come to realize the total vagueness of notions such as real numbers, sets, and countability, and moreover, that asking about the truth or falsity of the least upper bound property of the real numbers was as meaningful as asking about truth of the basic assertions of Hegel on the philosophy of nature.^[8] Any answer to such a question would be unverifiable, unrelated to experience, and therefore senseless.

However, within a few years Weyl decided that Brouwer's intuitionism did put too great restrictions on mathematics, as critics had always said. The "Crisis" article had disturbed Weyl's formalist teacher Hilbert, but later in the 1920s Weyl partially reconciled his position with that of Hilbert.

After about **1928** Weyl had apparently decided that mathematical **intuitionism** was **not compatible** with his enthusiasm for the phenomenological philosophy of Husserl, as he had apparently earlier thought. In the last decades of his life Weyl emphasized mathematics as "symbolic construction" and moved to a position closer **not only to Hilbert but** to that of

Ernst Cassirer. Weyl however rarely refers to Cassirer, and wrote only brief articles and passages articulating this position.

Quotes

Weyl's comment, although half a joke, sums up his personality:

My work always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually chose the beautiful.

The question for the ultimate foundations and the ultimate meaning of mathematics remains open; we do not know in which direction it will find its final solution nor even whether a final objective answer can be expected at all. "Mathematizing" may well be a creative activity of man, like language or music, of primary originality, whose historical decisions defy complete objective rationalization.

—*Gesammelte Abhandlungen*

The problems of mathematics are not problems in a vacuum....

[Impredicative definition's] vicious circle, which has crept into analysis through the foggy nature of the usual set and function concepts, is not a minor, easily avoided form of error in analysis.

In these days the angel of topology and the devil of abstract algebra fight for the soul of each individual mathematical domain. Weyl (1939b, p.500)

Topics named after Hermann Weyl

- See list of topics named after Hermann Weyl

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- ↑ *^a ^b* Newman, M. H. A. (1957). "**Hermann Weyl**. 1885-1955". *Biographical Memoirs of Fellows of the Royal Society* **3**: 305. doi:10.1098/rsbm.1957.0021 (http://dx.doi.org/10.1098%2Frsbm.1957.0021) .
- ↑ O'Connor, John J.; Robertson, Edmund F., "Hermann Weyl" (http://www-history.mcs.st-

- andrews.ac.uk/Biographies/Weyl.html) , *MacTutor History of Mathematics archive*, University of St Andrews, <http://www-history.mcs.st-andrews.ac.uk/Biographies/Weyl.html>.
4. ^ Hermann Weyl (<http://genealogy.math.ndsu.nodak.edu/id.php?id=7373>) at the Mathematics Genealogy Project.
 5. ^ Works by or about Hermann Weyl (<http://worldcat.org/identities/lccn-n50-18035>) in libraries (WorldCat catalog)
 6. ^ Vgl. Bernd Elsner, *Die Abiturarbeit Hermann Weyls*, in: *Christianeum*, Jg. 63, H. 1, 2008, pp. 3–15.
 7. ^ 1929. "Elektron und Gravitation I", *Zeitschrift Physik*, 56, pp 330–352.
 8. ^ Gurevich, Yuri. "Platonism, Constructivism and Computer Proofs vs Proofs by Hand" (<http://research.microsoft.com/~gurevich/Opera/123.pdf>) , *Bulletin of the European Association of Theoretical Computer Science*, 1995. This paper describes a letter discovered by Gurevich in 1995 that documents the bet. It is said that when the friendly bet ended, the individuals gathered cited Pólya as the victor (with Kurt Gödel not in concurrence).

Further reading

Primary

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- **Weyl, Hermann** (1939b), "Invariants" (<http://projecteuclid.org/euclid.dmj/1077491405>) , *Duke Mathematical Journal* **5** (3): 489–502, doi:10.1215/S0012-7094-39-00540-5 (<http://dx.doi.org/10.1215%2FS0012-7094-39-00540-5>) , ISSN 0012-7094 (<http://www.worldcat.org/issn/0012-7094>) , MR0000030 (<http://www.ams.org/mathscinet-getitem?mr=0000030>) , <http://projecteuclid.org/euclid.dmj/1077491405>
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- In connection with the Weyl–Polya bet, a copy of the original letter together with some background can be found in George Polya's article, "Eine Erinnerung an Hermann Weyl", p. 296–298. Polya, G. (1972). "Eine Erinnerung an Hermann Weyl". *Mathematische Zeitschrift* **126** (3): 296–298. doi:10.1007/BF01110732 (<http://dx.doi.org/10.1007%2FBF01110732>) . (Regrettably, this charming article was omitted from Polya's collected works.)

External links

- National Academy of Sciences biography (<http://www.nap.edu/readingroom/books/biomems/hweyl.html>)
- Bell, John L. *Hermann Weyl on intuition and the continuum* (<http://publish.uwo.ca/~jbell/Hermann%20Weyl.pdf>)
- Feferman, Solomon. "Significance of Hermann Weyl's das Kontinuum" (<http://math.stanford.edu/~feferman/papers/DasKontinuum.pdf>)
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Categories: 1885 births | 1955 deaths | People from the Province of Schleswig-Holstein | University of Göttingen alumni | ETH Zurich faculty | Institute for Advanced Study faculty | Foreign Members of the Royal Society | 20th-century mathematicians | Differential geometers | German mathematicians | German philosophers | German-language philosophers | Number theorists | Relativists

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Wikipedia® is a registered trademark of the Wikimedia Foundation, Inc., a non-profit organization.

Hermann Weyl (9 November 1885 – 8 December 1955) was a German mathematician, theoretical physicist and philosopher.. In chronological order. We cannot hope to give here a final clarification of the essence of fact, judgement, object, property; this task leads into metaphysical abysses; about these one has to seek advice from men whose name cannot be stated without earning a compassionate smile—e.g. Fichte. Hermann Klaus Hugo Weyl, better known as Hermann Weyl, was one of the most famous and skilled mathematicians of the 20th century. He was also popular as a philosopher and acclaimed for his contribution to theoretical physics, having published several works on space, time, matter, philosophy, and logic. He is also credited to have found out that by simply using group theory, most of the regularities of quantum phenomena on the atomic level can be easily understood. He was Hermann Klaus Hugo Weyl, ForMemRS (German: [vaˈɛːl]; 9 November 1885 – 8 December 1955) was a German mathematician, theoretical physicist and philosopher. Although much of his working life was spent in Zürich, Switzerland and then Princeton, New Jersey, he is associated with the University of Göttingen tradition of mathematics, represented by David Hilbert and Hermann Minkowski.