TRUMPET ACOUSTICS

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1973

It is not an easy matter to begin the writing of a chapter on the acoustics of a wind instrument for a book that is devoted to the history of the instrument. The problems of intelligibility can be difficult because we are in a territory where art, mechanical technology, and science are intertwined. However, these separate aspects may become more understandable if we travel back and forth between them, alternating descriptions of musical phenomena with their acoustical bases and accounts of scientific researches, sometimes starting with the historical origins of present-day ideas and sometimes using our current understanding as a basis for reviewing the labors of past workers.

The general shape of this chapter is strongly influenced by the vividly expressed precepts of the distinguished French acoustician, Henri Bouasse, who opened his two-volume work on wind instruments[1] with a chapter that is instructively entitled "*La science et l'archaeologie*." From the opening epigraph onward Bouasse warns us to go to the original sources, both scientific and aesthetic, and to strive always to keep our speculations within the bounds of the established knowledge of our time. To this end I have tried in this chapter to describe only those things that are within the realm of my direct personal observation, whether it is a matter of literary sources, musical phenomena, or scientific observation and calculation.

The chapter deals with the tone production process in the trumpet and with the general relation of the shape of the air column to the tone and to the musical response of the musical instrument to which it belongs. There is a continual alternation in this part of the chapter between descriptions of the practical behavior of the trumpet in the hands of a musician and the related acoustical concepts as they might be studied in the laboratory. Sometimes one of these is used for the vehicle for introducing a new concept or phenomenon, and sometimes the other. In similar fashion the history of some of these concepts is sprinkled through the rest of the discussion at points where it might become intelligible, or at points where it might itself serve as the entry door through which we may pass on our way to a new topic.

Acoustical Preliminaries

Musicians are always concerned not only with the correct tuning, but also with the clarity of tones they produce, and with the security with which they can start their sounds. They are also seriously interested in the flexibility of the dynamics available to them on their chosen instruments. For many years it has been possible to make fairly objective measurements of the intonation behavior of musical instruments, leading at times to an overemphasis on this aspect of musical performance. Tuning is a matter of musical context, so that to stand in front of a frequency-measuring machine is to play in an unfamiliar context to say the least. On the other hand, such observations can, when properly used, be the basis for great clarification of the practical business of obtaining musically correct intonation. For instance, if a player seeks out a way of blowing each particular note to give it the clearest sound and the fullest tone, it will automatically lead him to find the pitch that is most associated with the natural behavior of the instrument. In other words, the variability of the measurement of the player's pitch can be greatly reduced if we ask the instrument itself where it would, so to speak, best like to play. It is important to remember, however, that pitch is only one of the aspects of musical tone production to be considered, along with other subtleties such as clarity over a wide dynamic range and prompt starting behavior. It has not been possible until fairly recently to correlate such varying aspects of musical tone production with the details of instrument design.

At this point I would like to describe in a preliminary way some of the acoustical events that occur in a brass instrument when it plays any single note in its scale, and so to relate the playing properties of the instrument to the nature of its air column. We will find that very stringent requirements must be met by a musical air column if it is to play even one note properly; by a very fortunate circumstance, a design that "sings" well can generally also be made to play a whole gamut of tones in good tune. Thus our present restriction of the discussion to a single tone will not limit us in our eventual understanding of the complete instrument as it lives in an orchestra.

In essence, a wind instrument consists of a pipe or horn of varying cross section, which is coupled to a flow-control mechanism that converts a steady wind supply from the player's lungs into oscillations of the air column contained within the pipe. **Figure 1** shows in diagrammatic form the general structure that is characteristic of a brass instrument.

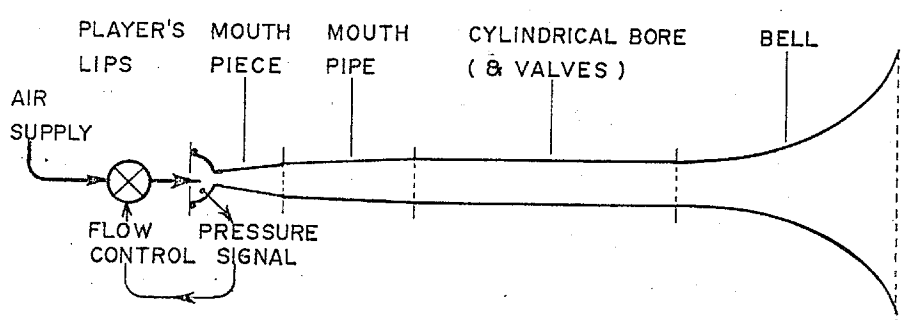

Figure 1: Basic structure of a brass instrument

The flow of air from the player passes between his lips, which open and close rapidly in response to the acoustical variations within the mouthpiece and so admit a periodically varying flow of air into the mouthpiece. It is this varying flow that maintains the oscillations of the air column. Putting it more concretely, the lips function as a sort of valve, which opens and closes in response to the oscillatory variations of air pressure in the mouthpiece as this pressure rises above and falls below atmospheric pressure. This pressure variation arises because the air within the instrument is swinging up and down its length. The air column, on the other hand, is maintained in its longitudinally swinging vibratory motion by the periodic puffs of air that are supplied to it via the lip-valve. If, for example, a trumpet is sounding the oboist's A (440 Hz), the air column is swinging back and forth along the horn in a complex motion that repeats 440 times a second, causing the lips to admit puffs of air of complex shape into the mouthpiece at this same rate of 440 times per second. It will perhaps help us to visualize what is going on if we turn our attention to a very close analogue of this air column system.

The "Water Trumpet"—An Analog to What Happens inside a Trumpet

Water waves moving in an open channel of varying cross section obey precisely the same equations as do the sound waves that oscillate in an air column of varying cross section. The channel is therefore an easily visualized model of the instrument's air column. The lengthwise swinging of air in the column is replaced by the lengthwise sloshing of the water in the channel, and our lips and lungs are replaced by a flow-control valve connected to the city water supply. **Figure 2** shows this hypothetical water trumpet, which is analogous to our familiar musical instrument.

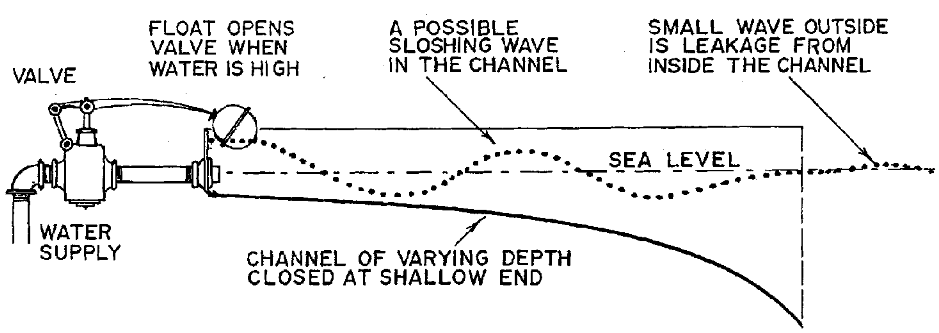

Figure 2: The "water trumpet

In this machine we will assume that a float valve is controlled by the varying height of the water at one end of a sloping channel, the other end of which communicates with the open sea. The valve is so arranged that it squirts a short burst of water into the channel whenever the water level here is high enough, and shuts off, or at any rate reduces, the flow whenever the water falls below this critical level. If one were to set up a sloshing of water in the channel, and if the sloshing were strong enough to trip the valve open at its peaks, the valve would be opening and shutting in time with the sloshing—a set-up that conceivably might maintain this back-and-forth swinging of water waves indefinitely.

By striking a skillet with a spoon, an iron bar with a mallet, a piano string with a hammer, or the end of an air column with a sharp slap of the hand, one can set into a complex vibration the skillet, the bar, the string, or the air column. This complex vibration is made up of a set of building-block vibrations, each of which has its own characteristic motion and its own particular defined frequency. The strength of the vibration of each of these vibrational modes depends on the place and manner in which the impulsive excitation is applied, as well as on its own vigor; but the frequencies at which these vibrations take place are solely determined by the object that has been struck. These characteristic frequencies, which are also called the natural frequencies of the object in question, are not necessarily members of a harmonic series, nor do they necessarily have any other orderly progression. These general remarks about natural vibrations apply perfectly well to the sloshings of water up and down our channel if they are all caused by a single impulse of water entering through a momentary opening of the valve.

Suppose now that in our initial investigation of the water trumpet, we select a particular taper for its channel and see whether it could properly instruct the valve (which fulfills the function of our lips on an actual trumpet) so as to make possible a sustained type of oscillation (such as is possible also when a bow interacts with a musical string, or when a woodwind player blows on his instrument). This sustained oscillation, which lasts until the player's lungs are deflated, is quite distinct in its nature from the impulsively started natural vibrations that must inevitably die away due to the effects of friction, as they do in the case of a struck skillet, bar, or piano string. Let our channel have a curving bottom arranged so that the water is very shallow at the closed ("mouthpiece") end where the valve acts, and fairly deep at the end that is open to the sea. Let us suppose for simplicity that the channel is of such length and depth that the natural mode of oscillation having the lowest frequency is one for which the sloshing recurs once each second. If this were the only mode of oscillation, it by itself would then be asking the valve to admit a burst of water at one-second intervals. So far so good, but what about the second mode of water oscillation it is possible to have in this channel? One might have (to choose a specific example) a channel having such rate of taper that the second natural mode of swinging takes place 2.25 times per second, so that it would call on the valve to inject pulses of water this many times per second. In a channel of this shape, the third mode of oscillation would like to keep things running at 3.58 repetitions per second, the fourth would prefer 4.87 per second, and so on.

The first injected pulse of water acts like a piano hammer to excite the sloshing modes characteristic of the water in the channel. The question then arises concerning the moment when the second pulse should take place if it is to keep all of these modes going. The top line of **Fig. 3** shows by black dots the instants, one second apart, at which the valve should open to sustain the lowest-frequency oscillation of the water. The second line similarly shows what is needed by the second mode, and so on. All the swingings are started together in our channel at the first burst of fluid injected, but they get into a quarrel very quickly about how soon the wave should inject its next little slug of water. We see that while mode 1 would like the valve opened after exactly 1 second, mode 2 votes to open it early at about 0.87 seconds, and mode 3 would prefer to have water injected at 0.84 seconds. Mode 4 is a different sort of troublemaker—it would be equally happy to have a burst quite early, at 0.82 seconds, giving a push to its fourth swing, or a trifle late at 1.02 seconds, in time with its fifth swing. Since all of these separate sloshings must cooperate in order to pile up the water high enough to open the valve, we find that our sloping water channel would not find it easy to maintain a steady oscillation. If on the other hand the float does not have to be raised too much in order to open it, then only a partial cooperation will be needed among the modes. Under these conditions a certain type of oscillation is possible in the channel. The system could find a workable compromise time at about 0.85 seconds, which corresponds to an overall repeating frequency of 1.18 sloshings per second. Interestingly enough, this frequency has no simple relation whatever to any of the channel's natural frequencies, although it is chiefly influenced by modes 2, 3, and 4. The oscillation is however sustained by a certain degree of cooperation among all of them.

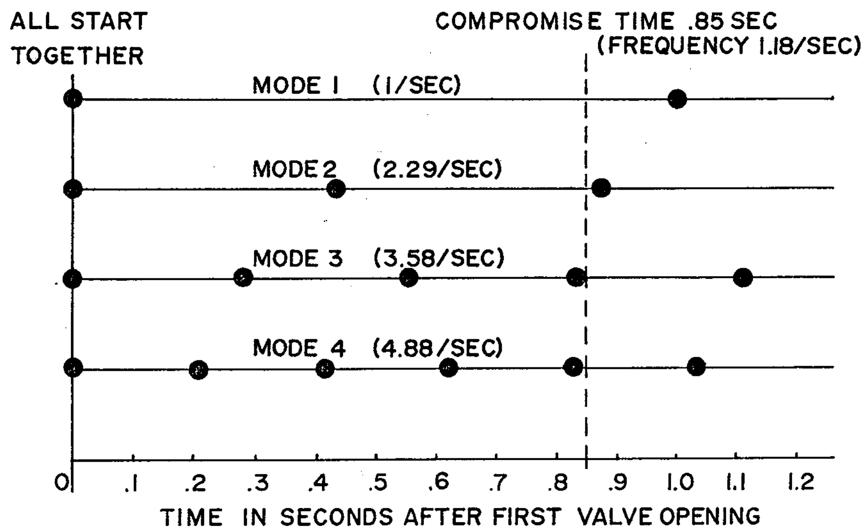

Figure 3: Schedule of valve openings for the first four modes

Another kind of oscillation that might be imagined in this water trumpet is one in which mode 1 swings in step with every fifth swing of mode 4, modes 2 and 3 being left out of the game. It turns out however that oscillations of this type are not usually possible because of certain anti-cooperative effects arising from the ignored modes. We find examples of all these phenomena throughout the world of wind instruments, and we shall meet practical versions of several of them in later part of this chapter.

Our introductory meeting with the sloshing modes of vibration of water in an open channel gives us an initial idea of the musical importance of the acoustical theory of waves in a channel of varying width, and of the technical understanding of the way a flow-control device can cooperate with an air column to maintain oscillations within it. Certain mechanical requirements must be met if sound is to be produced at all, and more stringent requirements are laid on us if we wish to produce dependable and pleasant sounds. The beginnings of a scientific understanding of these matters occurred many years ago, and we are in a position now to turn away from our metaphorical water trumpet and take up a few items of acoustical history. Let us begin with the flow-control aspect of sound production in wind instruments.

The Function of the Player's Lips

As long ago as 1830 Wilhelm Weber carried out experiments on the action of organ reeds which led him to a correct theory for the effect of a yielding termination (the reed, that is) on the end of a column of air.[2] In the present context this means that Weber provides us with an understanding of how the player's fleshy lips form a yielding closure to the mouthpiece, in addition to their special function as a rapidly acting flow-control valve. Hermann Helmholtz provided the next advance. In 1877 he added an appendix to the fourth German edition of his classic work, *Sensations of Tone*, which provides a brief but complete analysis of the basic mechanism whereby a reed, or the player's lips, responds to the acoustic pressure variations within the mouthpiece to control the air admitted from the player's lungs into his instrument.[3] The best account of the Weber-Helmholtz analysis and its musical consequences was made by Henri Bouasse in his book, *Instruments à vent*, the two volumes of which appeared in 1929 and 1930. These volumes contain what still constitutes one of the most thorough accounts of wind instrument acoustics dealing with the brasses and the orchestral woodwinds, as well as the flute and reed organ pipes. Bouasse has left us with a gold mine of mathematical analysis along with accounts of careful experiments done in collaboration with H. Fouché or selected from the writings of earlier researchers. The nontechnical reader can find an account of many of these matters in my book, *Horns, Strings and Harmony*,[4] and in more detail in a recent article in *Scientific American* magazine.[5]

The Function of the Pipe and Bell—Inside the Air Column

The tapering air column of the trumpet is the other partner in the collaboration that generates a musical tone. The history of our understanding of waves in tapered ducts (or "horns" as they are customarily called by acousticians) is a long one, and rather peculiar in that many times a basic understanding was gained and then lost, until a later researcher was led to rediscover the ideas all over again. On the other hand, a physicist who looks back over the history of his subject is struck by the prominent place that was originally occupied by musical acoustics. In fact it was one of the important sources of information about the nature of the physical world and a prime source of intellectual stimulation.

During the lifetime of Bach, the founding masters of theoretical physics took fourfold inspiration from the studies of the motions of the planets, the flow of heat, the flow of fluids, and, last but by no means least, the vibrations of musical strings and air columns. Already in the 1760s Bernoulli, Euler, and Lagrange succeeded in formulating the basic equation that enables us to make predictions about the behavior of sound waves in ducts of varying cross section. These early theoreticians discussed the behavior of sound not only in cylinders and cones but even in the family of so-called Bessel horns, to which we now know the trumpets are closely related. It is a curious quirk of history that this family of horns came to take its name from a nineteenth-century German astronomer simply because certain parts of the mathematical description of Bessel horn acoustics is based on mathematical results obtained in the course of purely astronomical calculations! Bernoulli and his contemporaries apparently did not consciously recognize that musical instruments of their day approximated the Bessel shape; this was simply the next shape following the cylinder and cone in the hierarchy of mathematical complexity.

This pioneering work, by men whose names are revered today by mathematicians, physicists, and engineers alike, lay buried for nearly a century. In 1838, the distinguished English mathematician George Green rediscovered the earlier results in connection with his studies of water waves in canals of gradually varying width and depth. This work by Green arose in response to a pressing practical problem, the erosion of the banks of England's transportation canals by waves set up by the canal boats as well as by tidal effects. It is in his work that we find justification for drawing an analogy between real trumpets and the water trumpet that was described in earlier pages.

In 1876 the German, Pochhammer, independently derived the equation and learned the properties of its most important solutions. In 1873 Lord Rayleigh published a brief paper on certain electrical phenomena in which he used a startlingly modern "operator method" of analysis that later he put to use in 1916 when he published a sophisticated and ingenious article on the acoustics of ducts of varying cross section. This paper included the derivation of the basic "horn equation" as an especially simple case. The implications of Rayleigh's 1916 paper have proved to be most helpful for some of us who have followed him. Finally, prehistory ends in 1919 when A. G. Webster published his derivation of the equation, and seemingly the world of science was ready to pay attention. Ever since, acousticians have referred to the basic horn equation as "Webster's horn equation," in defiance of its true history.

In the period that followed Webster, considerable practical use was made of horn acoustics in the design of phonographs and loudspeakers and for many other purposes. The subject of horn acoustics reached its contemporary maturity in the classic papers of 1946 by Vincent Salmon, whence has spring a spate of papers by many authors which has continued ever since. Readers wishing to become acquainted with the whole subject would do well to peruse the detailed and scholarly review article published in 1967 by Edward Eisner.[6] It is this paper with its extensive bibliographical commentary that I have used as a formal basis for my remarks in the preceding two paragraphs.

Let us digress here briefly to learn what is the nature of the Bessel horn shape and its relation to actual musical instruments. The mathematical formula that gives the diameter D of a bell in terms of the distance y from its large open end is

$$D = B/(y + y_0)^a$$

where y_0 and B are chosen to give proper diameters at the large and small ends, and a is the "flare parameter" that dominates the acoustical behavior of the air column. This parameter differs from one instrument to another, depending on its mouthpiece and leader-pipe design. Trumpet bells as far back as those made by William Bull[7] in the seventeenth century correspond closely to the shapes of Bessel horns having values of a that lie between the limits of 0.5 and 0.65. It is interesting to realize that the bell shapes that have evolved by the traditional combining of eye-pleasing artistry with practical experience are notably similar to one another in their acoustical description.

It is worthwhile to extend our digression enough to look briefly at the difference between loudspeaker horns and musical horns. The design requirements of a loudspeaker horn are of a sort that demand the best efficiency in radiating sound from a small source out into the air, whereas in musical instruments we will find quite the contrary requirements are laid upon the design—the bell flare of a brass instrument must be designed to save energy inside of the horn, giving strongly marked standing waves (sloshings of the air) at very well-defined natural frequencies.

Returning now to the musical side of "horn" acoustics, we find that Bouasse made essentially no use of the Webster equation. He gives an elegant and original derivation for it and solves it for Bessel horns and for the mathematically simpler exponential horns that find a certain application in loudspeaker design. Bouasse then drops the equation and makes no further reference to it.[8] In dealing with brass instruments, Bouasse restricted himself to an admirably clear exposition of the acoustics of what he called "cylindro-conical" composite air columns, which have been intensively studied by other as well, before and since. These air columns have however only a rough and qualitative acoustical resemblance to the musical brasses. Bouasse seemed to be quite unaware of the extremely important role played by the mouthpiece of a brass instrument in the overall fixing of the natural frequencies of the air column. This kept him from resolving many of the serious questions that he was however perceptive enough to raise. We will take up the subject of mouthpieces and their relation to the rest of the instrument at several points later on in this chapter.

Bouasse's contemporary, the British physicist E. G. Richardson, needs mention in our account chiefly because his widely read book, *The Acoustics of Orchestral Instruments*,[9] was the origin of a commonly held impression that trumpet bells are of what is known as exponential form. He also promulgated some peculiar notions about the flow of air in the mouthpieces of brass instruments. It is regrettable that such errors crept into the work of a distinguished scientist who made numerous contributions to other parts of musical acoustics.

An interesting document relating to the acoustics of brass instruments is an extremely detailed patent obtained in 1958 by Earle Kent of C. G. Conn, Ltd.[10] He achieved correct tuning by joining a sequence of "catenoidal" segments instead of using a single flowing Bessel-like shape. Segmentally proportioned bores are mathematically bound to give irregular intonation patterns unless they are counterbalanced by additional irregularities of taper or cross section. Practical examples of all these matters are thoroughly discussed in the patent, which also describes the way an electronic computer may be used to aid the design process.

One other worker who has been an active contributor to the science of brass instrument air columns is Frederick Young of Carnegie-Mellon University. He has published a series of significant papers beginning in 1960.[11] He represents the shapes of real brass instrument air columns by a cascade of very short segments, each with an assigned taper and flare. The smallness of the segments permits him to represent the properties of the smoothly varying horn with reasonably good accuracy because it avoids mathematically introduced irregularities of the sort that are deliberately accepted in a design (such as Kent's) based on the choice of a limited number of segments.

In 1970 William Cardwell obtained a patent for a particularly simple type of brass instrument design involving the ingenious use of a <u>single</u> segment of catenoidal bell, attached on the one side to a cylindrical main bore, and on the other to a short, rapidly flaring bell-end.[12] This design, which is somewhat related to that of Kent, was worked out independently, and proves very practical for the construction of higher-keyed instruments in E-flat and F. The two patents by Kent and Cardwell make interesting reading because of the insight they give into the practical problems of designing a brass instrument.

During the year 1967-68 Erik Jansson of the Speech Transmission Laboratory of the Royal Institute of Technology in Stockholm worked with me in Cleveland on a detailed study of air columns that are useful for musical instruments. This work, which was both theoretical and experimental, dealt with trumpet, trombone, and French horn bells. We unearthed a number of subtle relationships between our experiments and calculations that we could not clarify immediately. It is only recently that it has been possible to prepare complete reports on our results.[13]

An excellent source of information on brass instrument acoustics is to be found in the text and bibliography of the 1972 doctoral dissertation submitted by Klaus Wogram to the Technischen Universität in Braunschweig.[14] Of particular interest here is the extensive reference to European research, which is, unhappily, unfamiliar to many in the English-speaking parts of the world.

This completes our overall survey of the historical side of air column acoustics, so that we are in a position to return to the interrupted account of the way we have come to understand the means whereby the air column governs the player's lips to produce a tone.

The Cooperations Needed for Musical Results

For many years acousticians were puzzled and frustrated because their measurements of the natural frequencies in wind instrument air columns did not correlate very well with the pitches played by musicians on these instruments. As I have implied much earlier in this chapter in connection with the water trumpet, we now know that the musician's tone is sustained with the help of several natural vibration modes that form a sort of government-by-vote that we shall formally call a "regime of oscillation." This is a state of steady oscillation in which several air column vibrational modes collaborate with the lip mechanism to generate energy at several harmonically related frequencies at once.

There is abundant evidence that Bouasse was aware of the inadequacy of an oscillation theory based on the assumption that only one of the natural ("sloshing") frequencies of the air column is responsible for collaborating with the lip-valve to produce a tone. In other words, Bouasse recognized the inadequacy of the Weber-Helmholtz formulation of the oscillation problem even while he accepted its basic correctness. Bouasse's interest in the brass player's "privileged tones" (concerning which I will have more to say later) gives the clearest indication of this. Such concerns actually led him to describe the qualitative nature of the true collaborative state of affairs, even though he was unable to work out the quantitative relationships.

It was Bouasse's evident concern in these matters that provided the stimulus for the present author to take up a close study of the subject of sound generation in a system in which several modes of vibration collaborate.[15] The first fruits of this study were described in a series of technical reports commissioned in 1958 by Earle Kent of C. G. Conn, Ltd. These studies progressed with the aid of valuable counsel from many people. On the technical side I am particularly indebted to Robert Pyle, John Schelleng, and Earle Kent. In 1968 Daniel Gans and I reported on a more developed form for this theory that could deal with the interaction of several partials in a tone and gave an account of some of its consequences. We were even able to describe the successful design and construction of a nonplaying "tacet horn," which should have been able to sound, according to the Weber-Helmholtz viewpoint.[16] Since that time the work has been carried out much further here in Cleveland, particularly by Walter Worman who in 1971 presented a detailed report on it in the form of his Ph.D. dissertation.[17] For technical reasons his work was focused on clarinetlike systems, but the consequences are of general significance. Robert Pyle has presented results of related studies as his contribution to a symposium on brass instrument acoustics that took place in 1968.[18] Worman was able to trace out the ways in which a reed-valve interacts with an air column and showed that the particular 'playing frequency' chosen for the oscillation (along with its necessarily whole-number multiples) is one that maximizes the total generation of energy, which is then shared among the various frequency components in a well-defined way. The steady collaborative vibration belonging to a regime of oscillation is made up of the fundamental frequency component and a set of upper partials whose frequencies are exact whole-number multiples of the fundamental, whether or not the air column's natural frequencies are harmonically related. All that is required is that the natural frequencies are in sufficiently harmonic relation that they can set up a regime. The better that the lower two or three modes are in agreement with one another, the freer the speech of the instrument and the more centered its tone, in agreement with the observations of Bouasse.

It is time now to focus our attention on actual air columns of a musical sort, in order to understand the practical implications of the acoustical theory that we have merely sketched out so far. In the paragraphs immediately following, I will describe briefly certain laboratory measurements on musical instruments which can then be used as a basis for describing the tone that they produce. The ultimate goal of these descriptions is preparation for a meaningful discussion of the tonal similarities and differences to be found between the trumpet of today and of the Baroque era.

We have had hints already that the basic property of the horn that controls the vibration of the lips is the acoustic pressure developed in the mouthpiece cup under the stimulus of a given oscillatory flow of injected air. Let us see how this air column response might be measured in the laboratory independently of the complications engendered by the interaction of the air column with the player's lips. Conceptually, the simplest method would be to have a sort of oscillatory pump that feeds the mouthpiece cavity via a capillary tube such as one might cut from a hypodermic syringe (see **Fig. 4**). Sinusoidal (pure tone) pressure fluctuations that are produced at the motor's driving frequency in the pump cylinder give rise to a small, well-defined, and perfectly predictable oscillatory flow into the mouthpiece. If we then use a tiny microphone to measure the amplitude of the pressure fluctuations produced in the mouthpiece in response to the oscillatory flow of injected air, we will have the desired response information, and this could be displayed in the form of a graph as a function of pump driving frequency. As a practical matter one uses in place of the pump various cousins of the familiar loudspeaker. Such a driver is controlled by means of an auxiliary microphone that maintains a constant flow stimulus as one sweeps automatically through the interesting range of frequencies. Between 1945 and 1965, Earle Kent and his co-workers at C. G. Conn in Elkhart developed one form of this basic technique to a very high degree of dependability.

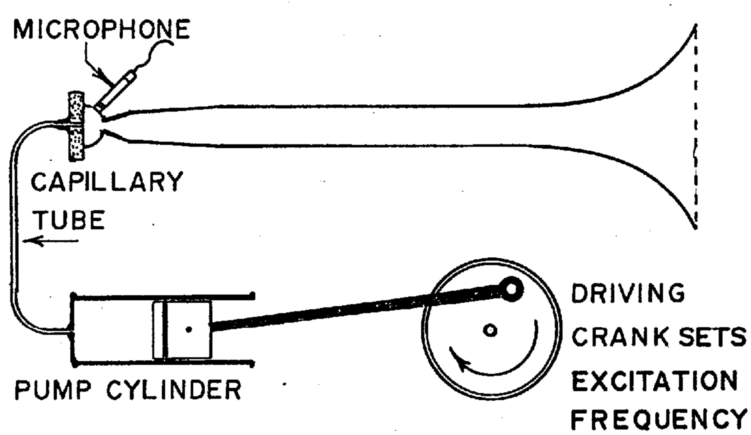

Figure 4: Measuring input impedance

There are several additional methods for measuring the pressure response of an air column to injected air flow. These are more subtle to understand, but they are sometimes freer of complications when making high-accuracy measurements. One such device of great versatility was first described by Josef Merhaut of Prague.[19] Another device that is of great utility for the study of brass instruments is an adaptation of an apparatus first constructed by John Coltman for his studies of the sounding mechanism of the flute.[20] There is yet another class of air column measuring techniques that is historically much older, being first devised by the Englishman, Blaikley, in the nineteenth century. A modern form of the Blaikley arrangement is easy to set up and involves measurements of the acoustic pressure variations in the mouthpiece, as before. However, the excitation of the air column is done by means of a properly monitored source loudspeaker placed near the open bell of the instrument, instead of through a fine tube leading into the mouthpiece cavity. In my laboratory I find that all of these techniques have virtues that adapt them particularly well to one sort of measurement or to another.

It is time to explore now what sort of pressure response curve we get as a result of a flow stimulus applied at the mouthpiece end of an air column. An acoustician would rephrase the question and ask for the input impedance Z of the horn as a function of frequency. When a piece of cylindrical trumpet tubing about 138 cm is attached to an excitation system, the pressure response curve shows dozens of input impedance (response) peaks whose frequencies are evenly spaced at odd multiples of about 63 Hz (see **Fig. 5a**). The nature of this pattern of pressure response peaks shows that they are to be identified with the "natural" frequencies of a cylindrical pipe stopped at one end that are described in every elementary physics textbook. Because the frictional and thermal losses of wave energy taking place at the tube walls increase with frequency, these resonance peaks become less and less tall at higher frequencies. The energy radiated into the room from the open end of such a pipe is, however, only a tiny fraction of one percent as compared with the energy that is dissipated at the pipe wall.

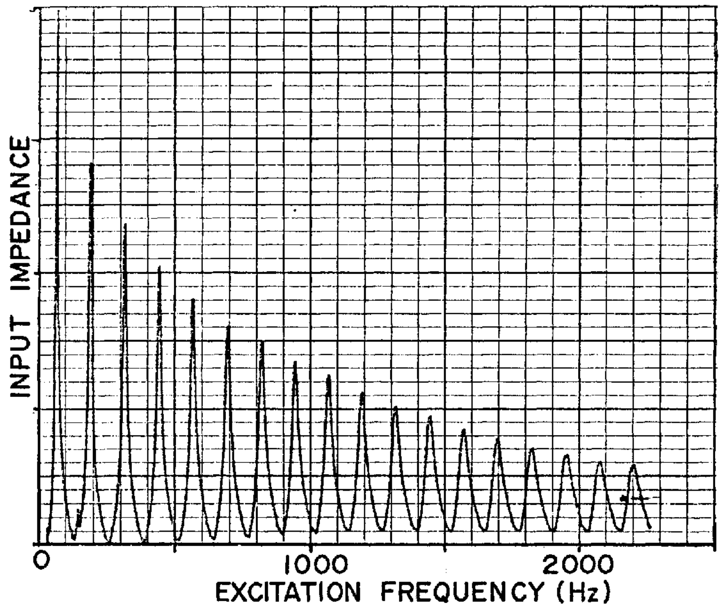

Figure 5a: Input impedance of a piece of cylindrical tubing

If we alter this piece of trumpet tubing by adding a trumpet bell, the input impedance curve changes to one of the sort shown in **Fig. 5b**. A close look at the frequencies of the response peaks shows that the first peak is hardly shifted by adding the bell, but the frequencies of the other resonances are lowered in a smoothly progressing order because of the way waves move in the bell. The trumpet-bell-plus-pipe system shows a rapid falling-away of the tallness of the peaks at high frequencies because an increasingly large fraction of the acoustic energy supply leaks out through the bell into the room. Above 1500 Hz there is essentially no returned energy from the flaring part of the bell. The small wiggles in the impedance curve at high frequencies are due chiefly to small reflections produced at the discontinuity where the bell joins the cylindrical tubing.

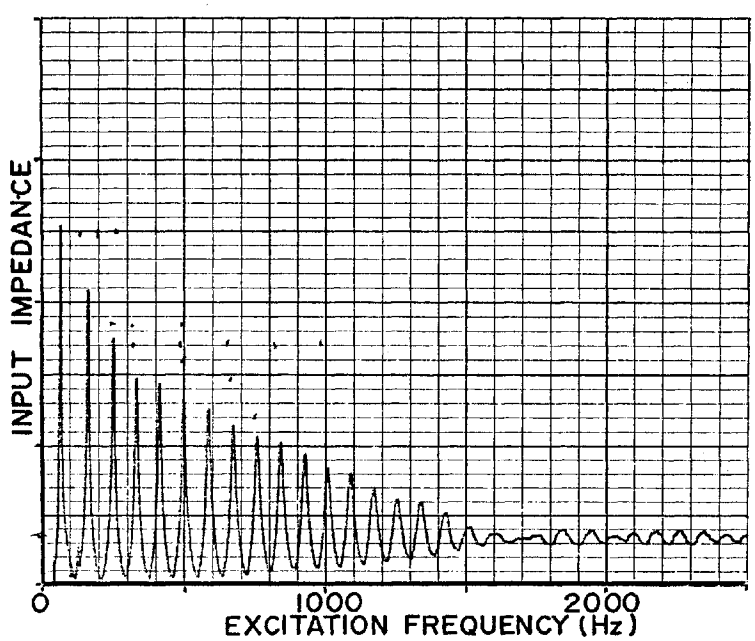

Figure 5b: Impedance modified by adding a bell

One need only glance at the impedance curve for a cornet (**Fig. 6**) in comparison with curves for a pipe or a pipe-plus-trumpet-bell to realize that the presence of a mouthpipe and mouthpiece has a considerable effect on the overall nature of the input impedance. The resonance peaks grow taller up to about 800 Hz, and then fall away in a manner that is only vaguely reminiscent of the falling away of the curves belonging to the trumpet bell plus pipe. The third and fourth impedance peaks of this particular cornet do not follow the smoothly rising trend that proves necessary for a really fine instrument. These irregularities of tallness are associated with irregularities in the frequencies of maximum response. They are caused by slight constrictions and misalignments of the tubing as it connects with the valve pistons, and with the junction of the main bore and the mouthpipe. One finds that irregularities of this sort give rise to difficulties in the tone and response of an instrument which are readily apparent to the player. The cornet whose response curve is shown here was made in 1865 by the respected British craftsman Henry Distin. The original owner of this instrument was Eckstein Case, nephew of the founder of what is now Case Institute of Technology of Case Western Reserve University. He gave it to Dayton C. Miller, also of Case, whose studies in musical acoustics in the early part of this century are well-known.

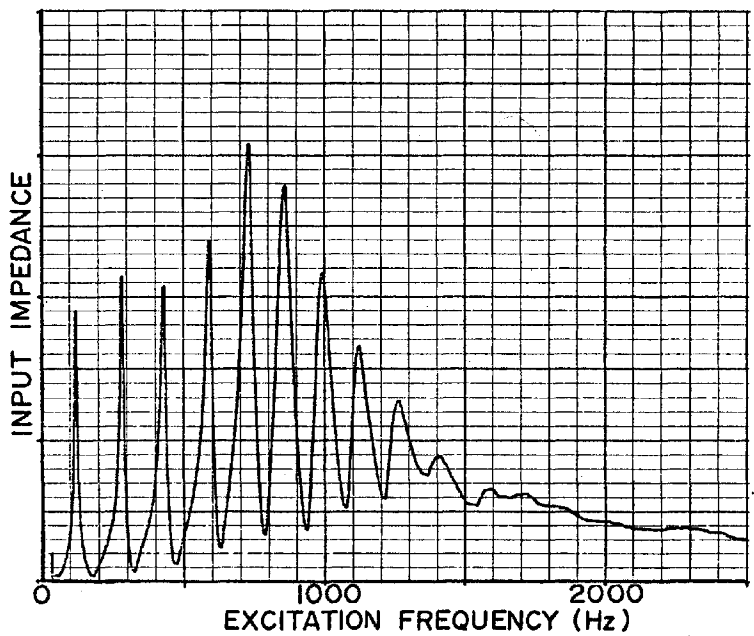

Figure 6: Measured impedance curve for a complete cornet

We have now had an introduction to the nature of the response curves that summarize the acoustics of trumpet-like air columns. We also have dealt in a preliminary way with the interaction of a player's lips with the air column of his instrument. We are finally in a position now to look at the nature of these collaborations between a player's lips and his instrument, as actual tones are sounded on a modern trumpet. First we will see how the tones are generated, and then we will look at the nature of these tones as they are played at various dynamic levels.

Figure 7 illustrates what goes on within a modern B-flat trumpet when the player is sounding the written note C_4 and the G_4 just above it. The regime of oscillation for the note C_4 is based on the second of the impedance maxima of the air column in consort with the fourth, sixth, and eighth of the peaks in the curve. When the tone is sounded at the pianissimo level, the playing frequency closely matches that of the second peak, which is the only contributor to the oscillation. As the loudness level increases, the other peaks successively become influential. A beginner attempting to play this note softly finds it to be quite wobbly because he is unable to maintain a steady lip tension, and the basic resonance of the horn for this note does not have a very large impedance. However, as he plays more and more loudly, the fourth, sixth, and to some extent the eighth peaks enter the regime one by one and add their stabilizing influence to the total oscillation.

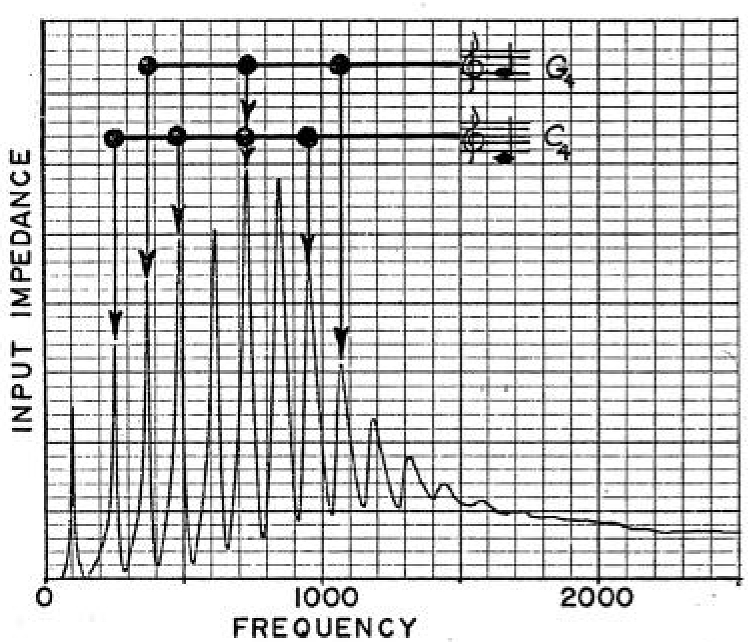

Figure 7: Impedance of a modern B-flat trumpet

When the player sounds the note G_4, the impedance maxima of the instrument that collaborate to form the regime of oscillation are peaks number three, six, and to some extent nine. For the note G_4 we observe that the impedance maximum that controls the pianissimo playing is much taller than it was for the note C_4, which makes the softly played sound more stable. As one plays somewhat louder, the very tall peak belonging to the second harmonic in the regime adds considerably to the strength and stability of the oscillation. For these reasons G_4 is one of the easiest notes to play on the instrument.

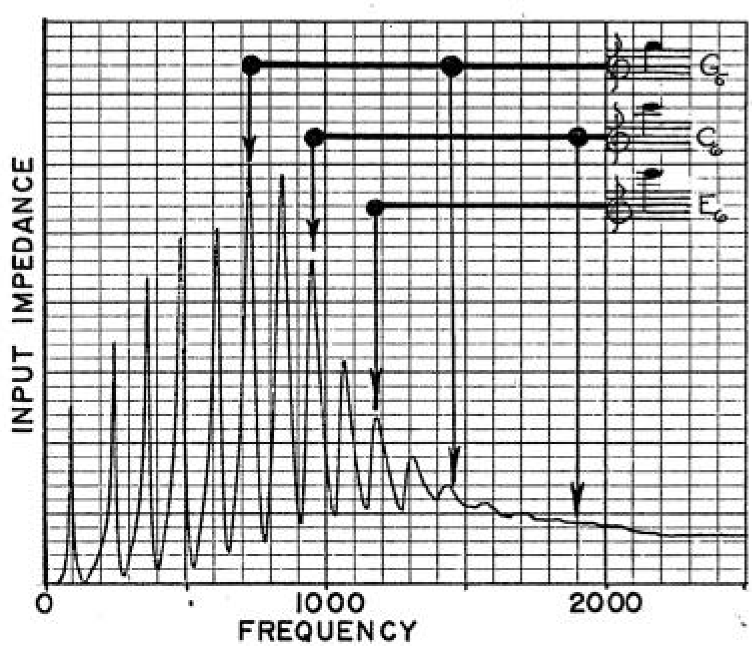

Figure 8: Decrease in impedance peak heights for higher notes

In **Fig. 8** we show once more the response curve for our trumpet; this time the regimes of oscillation are indicated for the written notes G_5, C_6, and high E_6. Notice that the G_5 is what might almost be called a solo performance—the regime of oscillation is dominated by the sixth impedance maximum of the instrument (which is a very tall peak indeed). Because there is only one impedance maximum contributing strongly to this oscillation, it is a note that is very well-described in terms of the original Weber-Helmholtz form of the theory, no matter what the dynamic level of the playing. The same remark applies to the C and the E above the G_5. However, these notes are more difficult for the player because the single active Z peak is not very tall. It takes an athletic trumpet player to play the high E and still higher notes. Quite aside from his problems with obtaining adequate lip tension, the player finds that the instrument has begun to turn into a megaphone in the range of such notes, and the energy production is almost completely due to the interaction of the air with the lips themselves in a manner quite analogous to the way the human larynx operates in producing one's voice. (On the Baroque trumpet the design of the bell and mouthpiece is such that the resonance peaks that help sustain these higher oscillations are appreciable and are active to somewhat higher frequencies than is the case on the modern instrument.

Let us look now at a pair of examples in which the player is able to produce a note on his instrument for a playing frequency that does not correspond to a natural frequency (frequency of maximum response) of the air column. Notes of this sort have been known to brass players since the earliest days, and were a part of the horn player's technique at the time of Mozart and Beethoven. The need for them was, however, reduced as the instrument because more mechanized. In recent years this type of note has returned to use, chiefly by musicians wishing to play bass trombone parts without the necessity for a special thumb-operated valve that is otherwise required. Tuba players also find the technique useful upon occasion. It is tones of this general class that attracted Bouasse's attention, and thence stimulated us to follow up their implications. These are the "privileged tones" referred to earlier. They are also sometimes called "factitious tones" by brass players, and are dealt with in a needlessly apologetic manner, as though there were something immoral about this manifestation of the complexity of nature! **Figure 9** shows the regimes of oscillation for two examples of these privileged tones. The written note C_3 in the bass clef, which is known to musicians as the pedal tone of the trumpet, is run as a regime of oscillation such that the 2nd, 3rd, and 4th resonance peaks of the instrument sustain an oscillation that lies at a frequency equal to the common difference between their own natural frequencies. There is actually a loss of energy at the fundamental playing frequency for this note, rather than a gain, because there an impedance minimum rather than a maximum in the response curve of the horn, which makes it possible to play in a stable manner only at a fairly loud dynamic level. Also one finds that there is a relatively small amount of fundamental component generated in the tone. This pedal tone regime will be recognized as being an almost exact analogue to the compromise frequency situation that we met much earlier in connection with our water trumpet. The situation for the written note G_3 is even more peculiar than for the pedal tone, in that the 2nd and 4th components of this new tone are the chief sources of oscillatory energy production.

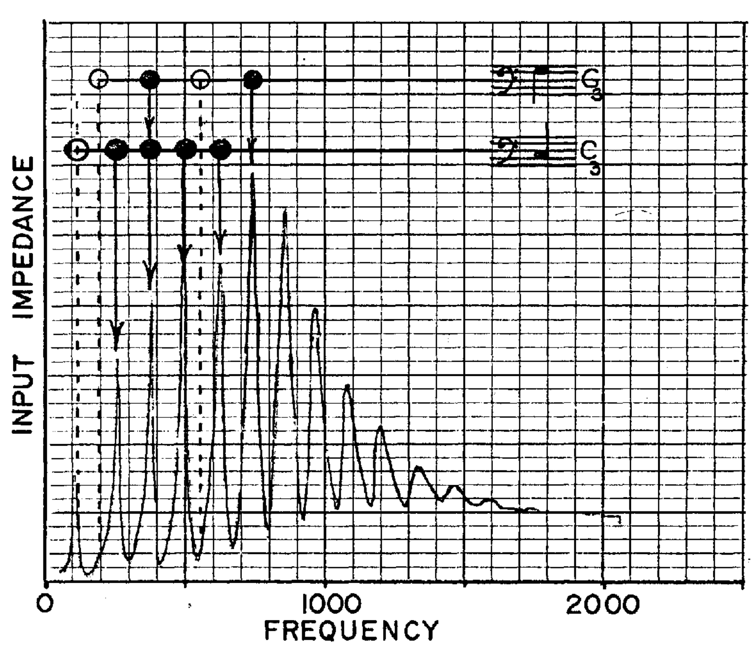

Figure 9: Regime of oscillation for two unvalved "privileged" tones

On the other hand the fundamental component of the tone and all the other odd-numbered harmonics do not contribute to the oscillation at all, because the air column's impedance is very low at those frequencies.

By now we have made a fairly detailed inspection of the ways in which a given air column (the open-fingered trumpet) collaborates with the player's lips to produce a set of tones. This set of tones (aside from the additional, closely related tone a musical fifth above the pedal note) makes up the harmonic series of pitches upon which trumpet music was originally based. The reader may be wondering what happens when any of the piston-valves are depressed on his trumpet. Nothing radically new takes place. The bell, mouthpipe, and mouthpiece design dominate the overall pattern, or the envelope, of a resonance curve—the pattern of peaks getting taller and taller as one goes from low frequencies to about 850 Hz and then falling away and disappearing at high frequencies. Because of this, the simple addition of cylindrical tubing into the middle of the instrument by means of piston-valves will merely shift the whole family of resonance peaks to lower frequencies, but will leave them fitting pretty much the same envelope. As a result, my earlier remarks apply essentially unchanged to all the in-between notes that are played using different valve combinations.

The Baroque Trumpet

We now turn our attention to an example of the earlier, valveless form of the trumpet, as we consider a 'Tarr Model' Baroque D trumpet made by Meinl and Lauber. **Figure 10** shows the ordinary input impedance curve belonging to this instrument when played with the vent hole closed. Comparison with Fig. 7 shows that the overall, qualitative shape of the resonance curves for the Baroque D and modern B-flat instruments are quite similar. The resonance peaks are, however, more closely spaced along the frequency axis for the D trumpet, simply because of its greater length. For ease in comparing the musical behavior of the two instruments, let us start by considering the regime of oscillation that supports the note concert A_3 of the D trumpet (at 220 Hz in modern tuning). This is only a semitone away from the open tone written as C_4 for the B-flat instrument, so that such matters as lip tension and the frequency response of our ears are roughly the same. When one plays loudly on the D trumpet, the tone A_3 is sustained by the cooperation of peaks 3, 6, 9, and 12, with some help from peak 15. All this is shown in the figure. The fact that the successive higher-frequency resonance peaks grow in tallness means that they keep their influence to a somewhat lower dynamic level of playing than is the case for their cousins (peaks 2, 4, 6, and 8) on the modern B-flat trumpet. This by itself gives the Baroque instrument a more steady A_3 than is the case for the C_4 of the modern instrument.

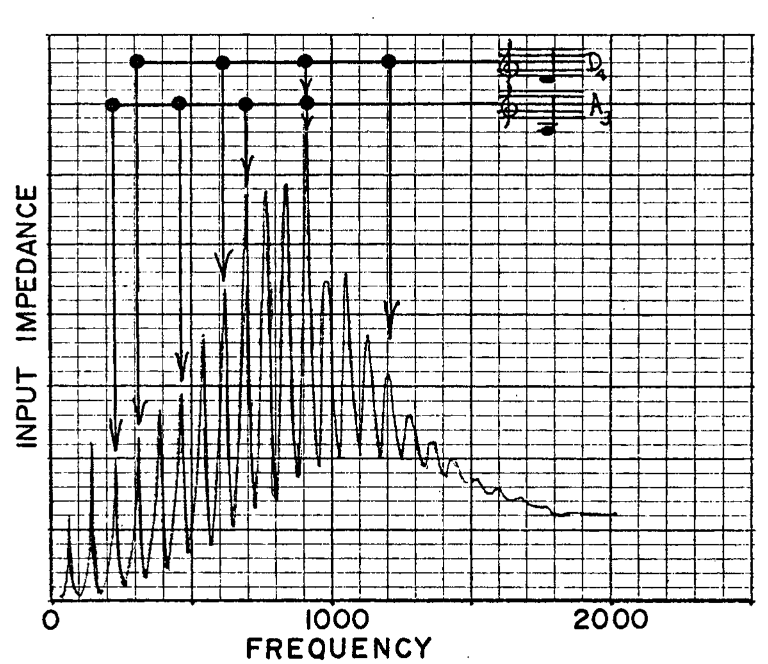

Figure 10: Impedance curves of a Baroque trumpet with its vent hole closed

The next member of the basic harmonic series of tones available to the Baroque musician is the note D_4 , which is sustained by a regime based on resonance peaks 4, 8, and 12, with some help from peak number 16. Once again we have a stable tone involving many cooperating resonances of the air column. The reader by now has enough knowledge of the dynamics of trumpet tone color that he can work out for himself the implications of the resonance curve for other tones in the musical sequence, using **Fig. 11** to tell him which resonances collaborate to produce the various tones. We note that above E_5 there is essentially no collaboration. We also note that everywhere in the scale the serial number of the tone is the same as the serial number of the tone in the musician's harmonic sequence. The seventh tone in this sequence, which is not customarily considered part of the named-note sequence, is a fairly well-supported tone based upon peaks 7 and 14. Peak 14 is located at a frequency that is slightly less than twice that of peak 7. When one plays softly (so that peak 7 dominates the regime), the tone comes out most naturally on our instrument as a slightly flat C_5. When, however, the dynamic level is raised progressively, the pitch drops toward a slightly sharp B_4 as peak 14 asserts its growing influence. In short, we have here a slightly unstable note that can be pulled up or down in pitch by the player to meet at least some of the musical requirements for notes written as B_4 or C_5.

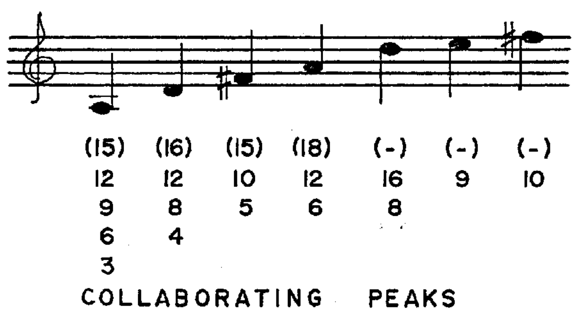

Figure 11: The collaborating resonances for various tones of a Baroque trumpet

The next in the series of tones available to the player is D_5. This is the highest of the tones in the Baroque trumpet's scale for which one has detectable cooperative effects from the higher air column resonances. Peak 8 determines the oscillation in soft playing, and the sound tends to run a little flat during a crescendo because peak 16 again has a frequency slightly lower than twice that of peak 8.

The Tarr Model Baroque D trumpet is supplied with a vent hole located at the junction of the bell with the main cylindrical bore. A considerable increase in the number of playable notes is provided by this hole, employing acoustical means that are not quite the same as those belonging to the unvented trumpet nor yet the same as those associated with note changes on a woodwind. In other words, it is not correct to think of the action of the hole as a simple 'cutting off' of the air column at its position. We are dealing with a tripartite air column—main bore, vent hole, and bell. Tonally we still have a close approximation to the normal trumpet in that most of the sound comes from the bell, and it is therefore radiated in a manner quite similar to that which characterizes the normal notes. **Figure 12** shows the measured resonance curve for our trumpet when its vent is left open. At first glance the curve appears very similar to that shown in Fig. 10 for the normal instrument. Closer inspection reveals, however, that the resonance peaks are spaced wider apart along the frequency axis, and also that the composite air column gives rise to a rather complex shape for some of the peaks, with small subpeaks and shoulders making their appearance here and there.

Let us look into the acoustical properties of several of the tones that can be played with the vented trumpet, beginning with the tone based on peak 3. This is a wobbly, unclear tone that comes out near C_4 when sounded pianissimo. Louder playing permits one to sound a tone as low as B_4 because of the influence of peak 5 on the second partial of the tone, and of peak 7 on the third partial. We also find it possible to sound the note as high as C_4#, when peaks 6 and 8 have supplanted peaks 5 and 7 as the influences upon the second and third components of the vibratory recipe.

The next tone in the series is a slightly sharp F_4# based on peak 4 when one plays softly, rising to G_4# when the tone is sounded strongly enough that peaks 8 and 12 begin to exert their influence. Above this we find a clearly defined A_4# based on peak 5. Even though peak 10 is quite sharp relative to twice the frequency of peak 5 it does not try to pull the note sharp on crescendo because of the presence of a small jog on its lower flank. This jog is located at exactly the right place for good cooperation with peak 5, so the note is quite stable.

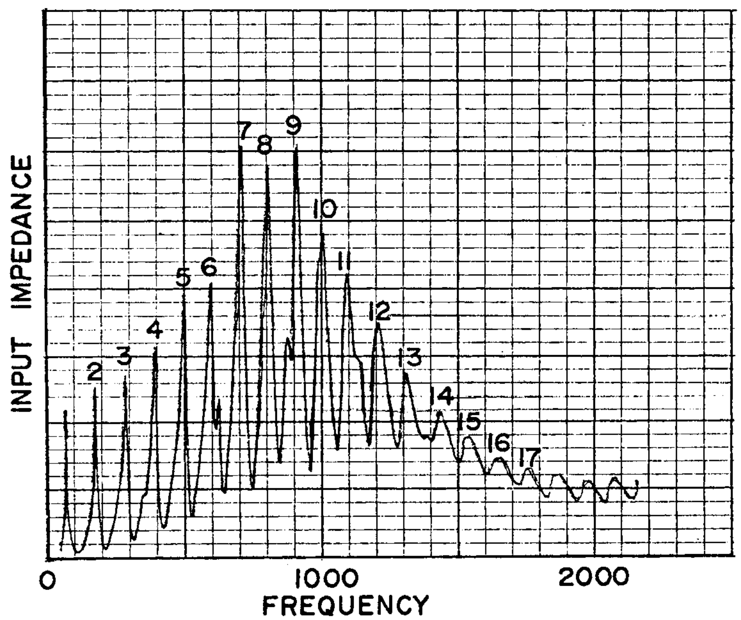

Figure 12: Impedance curve of a Baroque trumpet with its vent hole left open

Peak 6 of the vented series of resonances gives a slightly veiled C_5#that is pitch stable on crescendo but without any cooperative effects from the higher resonance peaks; the second partial of the tone falls at the dip between peaks 11 and 12, so that there is actually a certain amount of anti-cooperation.

The next note in our series is the tone D_5. This is not based on peak 7, but rather is a privileged tone whose frequency lies between those of peaks 6 and 7. The oscillatory energy is produced chiefly through the influence of peak 12 on the second partial component of the played tone, with a little energy input coming from the fundamental component as it works with the jog that is found on the high-frequency flank of peak 6. It will be convenient to label this privileged tone as 6-bis rather than as 7 in the sequence, to emphasize its special status.

We conclude our sketch of the oscillation dynamics of the vented tones with a remark that the seventh of the normal sequence here gives the note F_5, based on peak 7 with the help of peak 14, which pulls it a little sharp on crescendo. Tones 8 and higher run without collaborative influence from higher resonance peaks and need not be discussed further here.

The 'Internal' Spectrum of the Modern Trumpet

Our outline of the way modern and Baroque trumpets generate their tones through various cooperations between air column resonance peaks gives us a solid basis from which we can begin an inquiry into the acoustical and musical nature of these generated tones. Let us consider how tone color is influenced by the air column of the trumpet. We must first of all distinguish clearly between the internal tone color that could be perceived with the help of a probe microphone inserted into the mouthpiece cup and the tone color of the sound that issues from the bell in the normal manner. Our discussion so far has been concerned with the interaction of the internal sound waves with the lips, so we will begin with the tone color within the mouthpiece, and then consider how this is modified as it leaves the bell of the instrument and enters the concert hall.

Among the successes of Walter Worman's research into tone production by wind instruments was a clear-cut description of how the internal tone color (i.e., the strengths of the various partial components that make up the internal tone) depends on the playing level and on the nature of the air column. In instruments having a pressure-controlled air valve (whether lip, or reed), the strengths of the various harmonics generated in a regime of oscillation have a particularly simple relation. Leaving out the quantitative details, we find that theory and laboratory experiment agree on the following description of this internal sound. When one plays pianissimo on any note whatever, the internal sound has an almost purely sinusoidal waveform and sounds very much like a tuning fork if we listen in with the aid of a probe microphone. That is, the sound is made up of a fundamental component at the playing frequency, with almost nothing to be found at the frequencies of the harmonically related higher partials. As one plays louder the tone spectrum develops, harmonic by harmonic, the lower ones growing first. The tones that are generated by an oscillatory regime involving several resonance peaks develop their higher partials rather quickly and the internal sound becomes full and rich at a lower dynamic level than is the case for notes that rely upon only one or two air column resonances. We also find that the strength of any particular generated partial is large if the resonance peak associated with it in the regime is tall, and it is weaker if the peak is less tall.

Let us turn our attention to the practical implications of this for a modern trumpet. In 1970 a series of measurements was made with the help of Charles Schlueter, who plays principal trumpet in the Minneapolis Symphony (in 1970 he was a member of the Cleveland Orchestra). Schlueter played diminuendos and crescendos on one of my instruments, a B-flat trumpet, Selmer Paris, serial no. 4866. This was equipped with both an internal and an external microphone, which were fed to a tape recorder so that the results could be studied at leisure in various ways. The trumpet is the same one from which the resonance curves shown in Figs. 7, 8, and 9 were obtained.

Figure 13a shows the relative strength of the internal partials belonging to the written C_4 (see Fig. 7 for the air column resonances that participate in the generation of this tone). The uppermost of the Fig. 13a curves connects black dots that indicate the strengths of the first eleven partials produced when the trumpet is played fortissimo. The lower families of curves indicate similarly the strength of the partials at lower dynamic levels. We can see clearly here that as one plays more softly, the higher-frequency partials become weak more quickly than do the lower ones. At the weakest pianissimo that can be sustained by the player, the internal tone contains almost nothing beyond its fundamental component. I should like to emphasize that data of this sort are extremely stable. Measurements made on many separate tones, some starting *mp* and swelling to *fff* and some diminishing from *mf* to *ppp*, join smoothly when the curves are analyzed and the spectra plotted on a graph. The open-circled data points in our graph for C_4 can be used to show in addition that the behavior described here is determined chiefly by the trumpet and its mouthpiece, and only secondarily by the player. This particular curve shows the spectrum of a tone played at a loudness lying between *mp* and *p*. It is based on the analysis of a tone that I myself played and recorded in the course of setting up, testing, and calibrating my equipment, several days before Charles Schlueter came in for the more formal experiments.

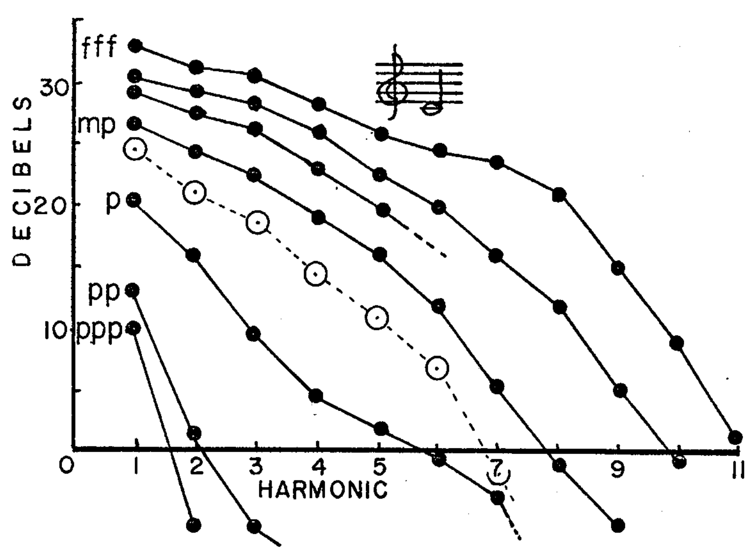

Figure 13a: Internal partials of the written C_4 (concert B-flat$_3$) on a modern trumpet

Figure 13b shows the measured internal spectra belonging to various playing levels of the written note C_5. We notice that the spectrum lacks appreciable strength in the harmonics beyond the fifth partial, even when the note is played very loudly. The tone does not become as pure at the pianissimo level, however.

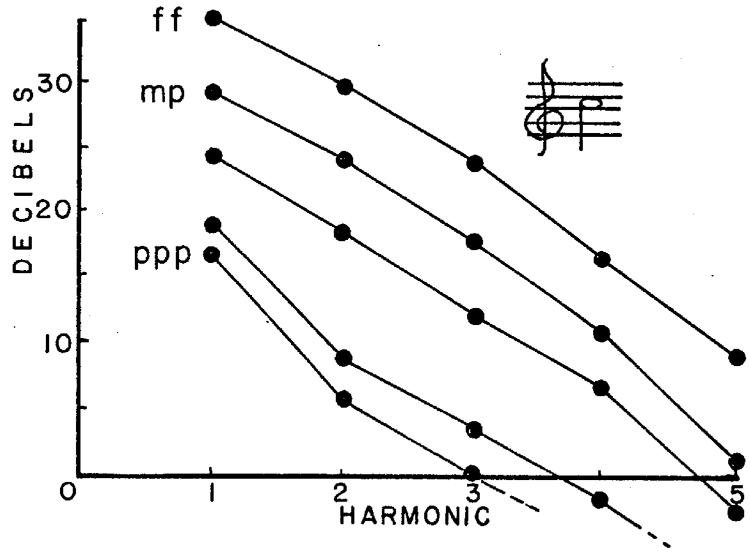

Figure 13b: Internal partials of the written C_5

Figures 14a and **14b** show similar spectra for the staff-top G_5 and the high E_6. The spectra of these higher notes show the progressive impoverishment of the tone color as one goes higher in the scale. It is possible to show by auxiliary experiments, especially with cylindrical pipes and with French horns, that these phenomena are not so much due to the limitations of the player's lips as they are to the diminishing number and strengths of the resonances that take part in the oscillations as one goes up the scale (see Figs. 7 and 8 for the regimes of oscillation that determine these tones).

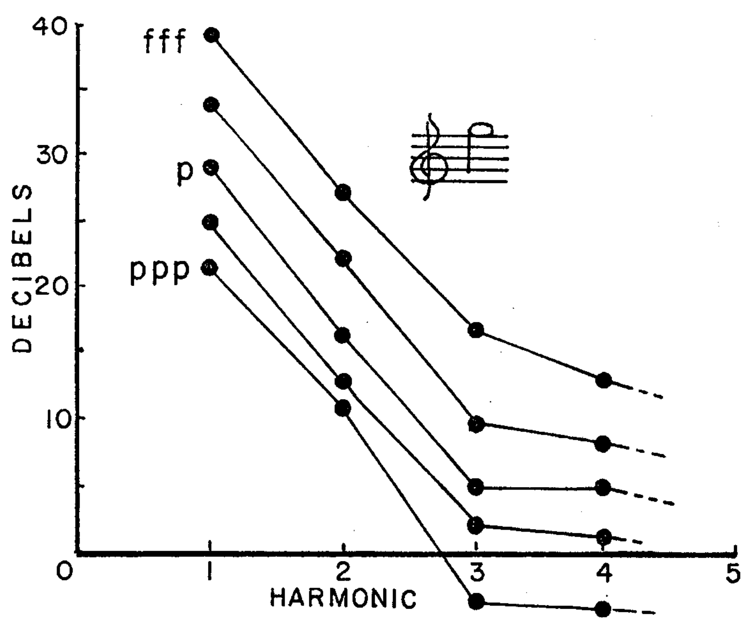

Figure 14a: Internal partials of the written G_5

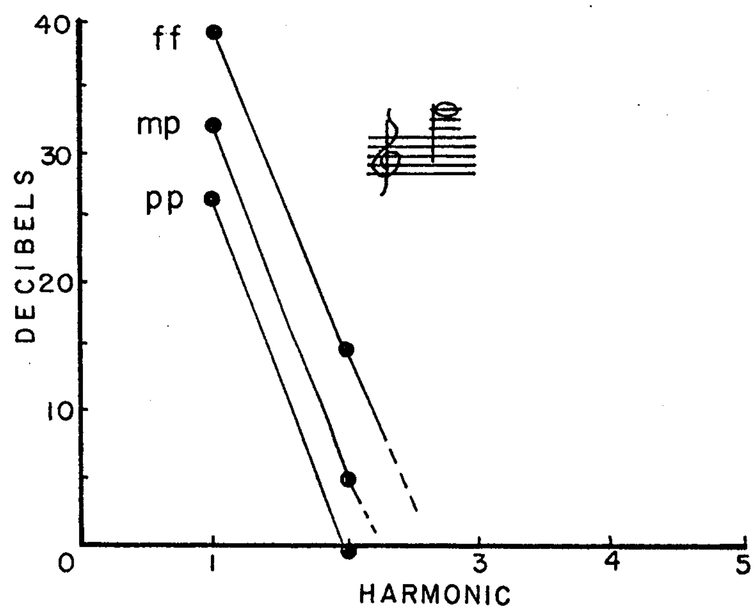

Figure 14b: Internal partials of the written E_6

I must emphasize at this point that the spectra described here have been plotted to include only the first few harmonic components of the tone. There are components at higher frequencies which have a noticeable effect on the overall tone color; however, they play a much subtler role than do the ones shown. One finds that the basic character of most wind instrument tones is established by the first half dozen components, to the extent of identifying the instrument's style of construction, its player, and often even its maker. I should also remind the reader at this point that the internal spectra discussed so far are not the ones that are supplied to our ears in the concert hall.

The 'Internal' Spectrum of the Baroque Trumpet

Before considering the tone of a modern trumpet in the concert hall, let us turn our attention to the nature of the internally measured spectrum of various notes on a Baroque D trumpet replica. A week after the series of measurements on the modern B-flat instrument, Charles Schlueter and I joined forces to make similar observations on the Tarr Model replica that we discussed earlier. **Figure 15** shows the strength of the partials of concert A_3 played at various dynamic levels. Because the pitch of this note is only a semitone below that of the written C_4 played on the modern trumpet (see Fig. 13a), we may assume that the player's lip tension, etc., is roughly comparable on the two instruments. A superficial comparison of Fig. 15 with the corresponding part of Fig. 13 does not give an impression of great difference between the two spectra. Closer inspection shows, however, that at all playing levels the third partial of the Baroque instrument is somewhat stronger relative to its brothers than is the case for its modern counterpart.

Analysis of the higher tones played on the Baroque trumpet gives results that are quite reminiscent of those obtained for the modern instrument. However, the increased strength of the second and third partials relative to that of the fundamental component in the Baroque instrument becomes even more pronounced for the higher notes than it is for the lower one upon which we have already remarked.

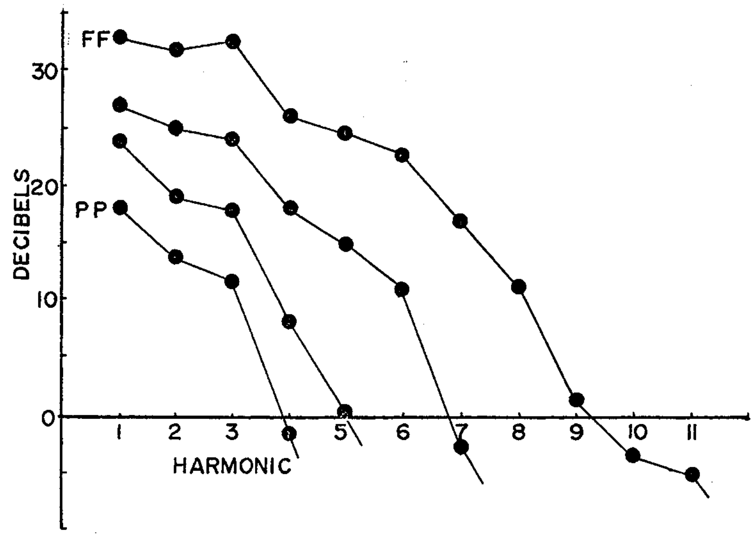

Figure 15: Internal partials of concert A_3 played on a Baroque trumpet

Relation of Internal to External Tone Color Spectrum

Up until now we have been discussing the strength of the various harmonic components of a tone as they are measured inside of the mouthpiece by means of a special microphone. What one hears in the concert hall is of course a very different thing. The spectrum generated inside the mouthpiece is transformed into the spectrum found in the concert hall by the selective nature of the transmission of sound from the mouthpiece out through the bell flare into the room. There are many facets to this transmission process, even without taking into account the complexities of room acoustics or the complication of our perceptual mechanism, which does a remarkable job of processing the great irregularity of room properties to give us definite impressions of the tone quality of our instruments. The qualitative nature of the transformation of the spectrum from inside the mouthpiece to that heard in the concert hall is similar to that of the treble boost control of a hi-fi amplifier. In other words, the higher components of the internally generated tone are preferentially radiated. We are only just beginning to unravel the mysteries of the transformation function.

Studies of the closely related but much simpler transformation between the internal spectrum and one obtained from a microphone located at a carefully chosen spot immediately outside the trumpet bell (as was the case in my experiments with Charles Schlueter) show that the transformation differs between loudly played and softly played notes, and that it differs from player to player much more than is the case for the internal spectrum itself. For this reason, I show only the general, qualitative nature of the transformation function in **Fig. 16**. It is also because of this only partially resolved complexity that I will confine my discussion of the general nature of the external spectrum to a single example, and use it as a basis for only such comments on its musical implications as are scientifically justified at the present time.

Figures 17a and **17b** show the external spectra of the written C_4 (concert B-flat) played on the modern B-flat trumpet, and the concert A_3 of the Baroque D trumpet. These spectra are analyzed from recordings made with an external microphone, recorded on the second track of the same tape that carries the internal spectrum data from which are derived the curves shown in Fig. 13a and in Fig. 15. We can see at a glance that the external spectra produced by the two instruments are quite different. The modern trumpet has a loud-playing spectrum in which at least the second and third components are stronger than the fundamental component, whereas, in the spectrum of the Baroque instrument, all the lower partials are of more or less equal strength. During a diminuendo the spectrum, and hence the tone color, of a modern instrument changes (in the direction of simplicity) considerably more than does that of the Baroque instrument. In other words, during a decrescendo the Baroque instrument keeps its fully developed tone down to a much lower dynamic level than does a modern trumpet. The implications of this difference

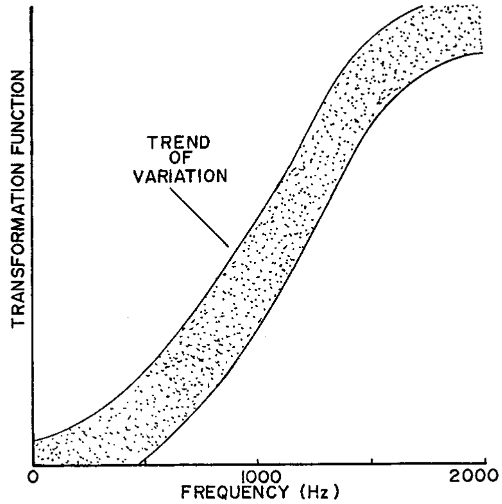

Figure 16: Generalized transmission function showing the variation of acoustic energy with frequency

for the musician are considerable, especially when account is taken of a closely related acoustical distinction between the two designs. It is inherently easier on a long-bore instrument to achieve the accurately aligned resonance peaks that give stability of speech to the mid- and low-range tones of a brass instrument. For this reason a really good Baroque instrument permits comfortable playing at low dynamic levels and allows the spinning-out of diminuendos to an extent that can be quite astonishing to the player of present-day trumpets. These advantages of the design of the Baroque instrument explain why it is so difficult to maintain tonal and dynamic balance between a modern trumpet and a small chamber group of the sort called for by Bach and Handel. The player cannot 'lean into' his instrument for full tone and security of attack without drowning out everyone else. I leave a detailed discussion of these matters to other contributors to this book and will merely remark that Philip Bate's brief but well-chosen words on this subject[21] are in entire accord with the acoustical facts described above.

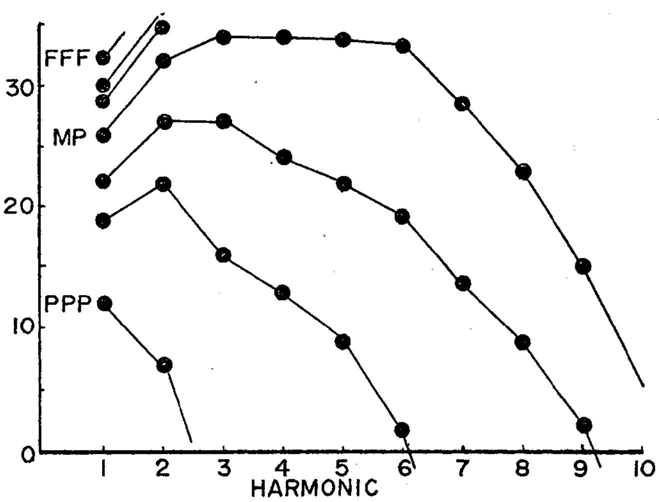

Figure 17a: External spectra of written C_4 (concert B-flat$_3$) on a modern B-flat trumpet

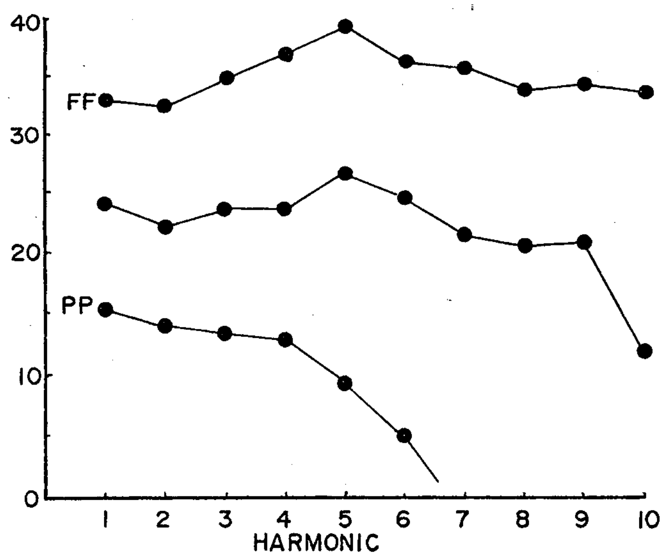

Figure 17b: External spectra of A_3 on a Baroque D trumpet

The Menke Trumpet

At the end of Bate's discussion he alludes to the work of Werner Menke of Leipzig, who in the 1930s strove to devise a trumpet combining the tonal advantages of the old trumpet with the technical convenience of modern valve mechanisms. My own interest in the acoustics of brass instruments stems from a reading in 1948 of Menke's book[22] describing his researches. Musicians who have played on examples of the Menke design (as constructed by Gebr. Alexander in Mainz) are unanimous in their opinion that the instrument is extremely hard blowing, with the high notes almost unplayable. These faults seemed to me to be inherent since they are present despite the perfection of workmanship characteristic of everything made by the Alexanders. I found it curious that a two-valved F trumpet using bell and mouthpiece of reasonably familiar design could be so recalcitrant in its behavior. In 1969 I had the privilege of visiting Anton Alexander in Mainz, which provided an excellent opportunity to see a Menke instrument and to learn of its properties at first hand. Through the kindness of Mr. Alexander, I have had this instrument and a matching bell with me in Cleveland on extended loan for study and experimentation.

The regularity of the peaks of the measured input impedance curves of the Menke trumpet shows its careful workmanship. We are provided here with a beautiful example of the way in which resonance peaks that are placed accurately enough by ordinary tuning standards may not be suitably arranged for good cooperation in the various regimes of oscillation. The long cylindrical bore does not work well with any conventional mouthpiece. However, recently I have found it possible to construct a mouthpiece that suits the instrument reasonably well over the whole of the range intended for it by Menke. Presumably further work will perfect the mutual adjustment of mouthpiece and air column. It is certain already that Menke's musical intentions were entirely correct and compatible with the basic principles of acoustics.

The Problem of Clean Attack

So far we have explored the acoustical requirements that must be met by a trumpet if it is to play steady tones one by one. I have also implied that in general an instrument that speaks with stability and clarity will also have a good tone, in the musician's sense, and also that it can be built to play accurately in tune. There is one more attribute that is required of a musical instrument if it is to be considered of first quality: it must start its tones cleanly and promptly and be forgiving of small inaccuracies of the player's lip tension as he shifts rapidly from one note to the next.

The acoustical properties of an air column that contribute to a clean attack for every note may be looked at in two parts. To begin with, anything that makes for a 'happy' collaboration during the time a note is sustained will in general contribute to a prompt build-up of an oscillation when the tone is started. It seems almost to be a truism that the placement of resonance peaks at frequencies that favor the maintenance of oscillation should also favor the secure beginning of such an oscillation. For the woodwind player, this is very nearly the complete story. The brass player has, however, one more thing to contend with because of the great length of the instrument with which he produces the same notes as the shorter woodwinds do. Basically he must deal with the fact that in a long instrument it takes a long time for acoustical 'messages' to travel from mouthpiece to bell and back, informing the lips of the collaborative job they must do with the air column.

Let us see what happens to the initial part of the acoustical disturbance set up by the player's lips as he attempts to start a tone. This disturbance travels down the bore with a speed that depends on the rate of flare of the air column, and then in the flaring part of the bell some of this wave is reflected back toward the mouthpiece, with the remaining (generally small) fraction of it being radiated out into the room, where we can hear it. The reflected wave, upon returning to the mouthpiece, 'tells' the lips how and when they must reopen to admit the next puff of air in the sequence of puffs that sustain the tone after everything has settled down. Until the first reflections begin to come back, the lips are on their own. The air column has not yet expressed its preference for one or another of the frequencies with which it is able to collaborate. A superb player of the sort who has learned to buzz his lips in perfect pitch even when playing on a brass ring has little problem with a good trumpet. He buzzes the desired note and the air column is very happy to collaborate in its own good time. The collaboration cannot of course even begin until there has been time for at least one complete round trip of the initial sound to go down the air column and back, and it requires two or three more round trips before the regime of oscillation has set itself up completely. In a fast, running passage, there is barely time for one regime of oscillation to be set up before it must give way to the next.

We are now in a position to understand why small irregularities of either the cross section or the taper angle in the air column can be fatal to a clean attack. Even a small step change in cross section in the middle of the air column (produced by a tuning slide, etc.) or an ill-chosen change in the taper will return a premature echo of significant size to the mouthpiece, an echo that is not even a replica of the original disturbance. Such ill-timed, ill-shaped return echoes can upset the best-trained of lips and, having spoiled the steadiness of this initial vibration, will ruin the attack. Such irregularities are of course a complete disaster for the less-skilled player, even if he can maintain a good sound once it is started. Curiously enough, it is possible to build an instrument that gives a strong, clear sustained note, and yet is unwilling to start well at all. Various discontinuities may deliberately be introduced to offset one another, or to counteract other faults of the air column. This can give a well-tuned stable instrument, with good tone, even though it can be extremely treacherous during the attack of its various notes. Every musician has met such instruments, as well as those that attack cleanly but which lack other virtues that are needed for the production of real music.

My description of the starting-up process for trumpet tones so far has left the impression that the longer the air column, the slower or more risky the attack necessarily must be. Anyone acquainted with the long Baroque trumpet is aware, on the other hand, that these instruments are at least the equal if not the better of the modern short design in starting characteristics. The explanation for this can be found in terms of an understanding that 'the speed of sound' is a phrase that actually refers to two different aspects of sound propagation.

The first sort of speed that turns up in relation to acoustical disturbances is what the physicist calls the 'wave velocity' for a sound of precisely defined frequency. The wave velocity in general depends jointly on the frequency of the sound and on the rate of flare of the air column through which it travels. We find that it is this wave velocity for sounds in an air column that determines its resonance frequencies. In the special case of straight-sided air columns (the ordinary pipe, and the simple cone), the wave velocity is independent of frequency, and becomes equal to what we may call the ordinary speed of sound in the open air.

The second kind of speed is known as the 'group velocity.' We may say that this is a measure of the speed with which abrupt disturbances travel down an air column. The group velocity depends on the frequency component that predominates in the disturbance and, once again, it depends on the rate of flare of the horn. As above, the group velocity for sounds in a straight-sided air column is independent of frequency and is equal to the open-air speed of sound.

The actual values of the group velocity and the wave velocity most closely related to it are not at all the same in most hornlike air columns. The round-trip time for the initiating disturbance of a trumpet tone is calculated from the group velocity and not from the wave velocity. In other words, the trumpet maker has the very interesting technical problem of getting a whole set of wave velocities to come out right if he wants good steady sounds, and he must at the same time achieve a correct set of group velocities if he wants these tones to start cleanly!

Now we can understand why the modern short trumpet does not necessarily have an advantage over its longer ancestor. Its shortness does give it an initial advantage, but even in its valveless form we find it more difficult to find a bore shape that simultaneously reconciles the group and wave velocity requirements over its playing range than is the case for the long instrument playing the same notes. The chief reason is that it becomes easier to reconcile our twin requirements for the higher members in the series of natural frequencies belonging to a given trumpet. It is here the longer trumpet gets the advantage for notes in the treble clef and above. Furthermore the presence of sharp jogs in the valves and sharp reversals of the bore (even when the cross section is maintained) produces troublesome early echoes in the modern valve trumpet to the extent that individual members of the tribe may or may not surpass individual members of the older group.

Mahillon in Retrospect

Now that a fairly thorough outline of our present-day knowledge of the basic acoustics of the trumpet has been presented, we are in a position to consider retrospectively one of the leading figures in the nineteenth-century development of brass instruments, Victor Mahillon, whose book, *Éléments d'acoustique musicale et instrumentale*,[23] has been very influential. To begin with, we must remember that Mahillon was an extremely skilled craftsman, possessed of wide experience, great ingenuity, and a perceptive ear. He did not have an extensive formal scientific training but was eager to use whatever scientific knowledge that was available to him in the furtherance of his goal, the improvement of musical instruments. What we must understand, however, is that the established body of science available to Mahillon could not help him much beyond providing guidelines and suggesting the methods of systematic research. Mahillon presents an admirably clear description of the nature of standing waves in a cylindrical tube that is open at both ends and for a pipe closed at one end. He points out that the flute plays at frequencies that are related to the resonances of the doubly open pipe and that the reed instruments and brasses run at or near the frequencies characteristic of the pipe stopped at one end. Reference to Fig. 5a shows that for a cylindrical pipe these frequencies favored for collaboration with the lips form the odd members of a harmonic series. In other words the musical interval from the first to the second regime of oscillation of a cylindrical pipe is a musical twelfth. Furthermore the interval from the second to the third regime of oscillation is a major sixth. Clarinetists are completely familiar with this pattern of behavior, and the brass player can observe the same pattern in a moment if he will play on a length of cylindrical pipe (be sure not to use a mouthpiece).

Mahillon was obviously aware of the fact that enlarging the upper (mouthpiece) end of a stopped-pipe wind instrument lowers the frequency of the first resonance mode and stretches the musical interval between the playing regimen, and that conversely a progressive enlargement of the lower half of the cylinder raises the first natural frequency and compresses the musical interval between the successive playing regimes. While Mahillon does not state these matters explicitly, the nature of the rules he gives for laying out the tone holes of a clarinet shows that he has allowed for these effects in a real clarinet. Every clarinet maker learns very early how to adjust the twelfth-shrinking effect of the bore enlargement in the lower joint to compensate for the stretching effect of the necessarily compromised register hole position.

Let us digress for a moment to see the implications for brass instrument design of my remarks on the effect of bore enlargements on a cylinder. We have seen that the cylinder gives a series of odd harmonics for its sequence of tones when played with a pressure-controlled lip reed. We might conceive of enlarging a pipe at its lower end and reducing its size at the mouthpiece end just the right amount that the original interval of a twelfth between the first and second regimes is contracted to become an octave, with the interval between regimes two and three reduced from a sixth to a fifth. In other words, can we arrange a flaring air column such that when it is closed at the small end, its natural frequencies form a complete harmonic series? The answer is a qualified yes. Figure 5b shows that the addition of a bell to a cylindrical pipe (corresponding to an enlargement at the open end of an extended cylinder) produces changes in the desired direction. Figures 6 and 7 show that the further addition of a proper lead-pipe and mouthpiece will put all but the first resonance peak into a harmonic relation. The first peak, however, remains low by about 30 percent. We have already seen that the instrument plays a harmonic series by ordinary means for all but the pedal tone. We have also seen that the pedal note sustains itself in a different way, but at a frequency that is exactly where our idealized horn would put the first resonance peak.

In the light of the last paragraph, we can see why Mahillon was led (along with many people since) to treat the trumpet as a doubly open cylinder when in acoustical fact it is a modified stopped pipe. In his introductory chapter on brass instruments Mahillon speculated briefly on the nature of a suitable air column shape. He wrote, "[I] have the conviction that [the] proportions follow . . . a geometric curve whose form approaches that of a hyperbola."[24] This is in fact an entirely correct surmise, since the hyperbolic shape is that of a Bessel horn with a flare parameter equal to unity. Mahillon thus clearly recognized the basic nature of a noncylindrical brass instrument air column. He did not comment on the apparent conflict in his discussion when he gives a formula for the frequencies of a brass instrument in terms of its length; this formula is formally identical with that for a doubly open cylinder. In modern notation it may be written,

$$f_n = nc/2[L + 2D] \quad n = 1, 2, 3, \ldots$$

Here f_n is the frequency of the n'th member of the harmonic series and c is the speed of sound in free air. The length L is measured from the mouthpiece rim to a somewhat ill-defined point near the end of the bell where the diameter is D. The shape of the formula tells us symbolically what he also said in words: the quantity 2D is taken to be a sort of 'open-end length correction' for the bell. I must emphasize that the formula is based not on mathematical physics but on the results of practical experience. Furthermore, it is important to understand that the relation of this formula to the properties of a doubly open cylinder is purely metaphorical and has no visible connection with Mahillon's undoubted understanding of the stopped air column of varying cross section. Nevertheless, despite its nonexistent mathematical basis, Mahillon's empirical formula has successfully guided instrument makers of several generations in their computations of horn lengths and the calculation of valve crooks.

From a strictly utilitarian standpoint Mahillon's success has been of mixed benefit. It permits the easy calculation of a workable trumpet of conventional design, which is to the good. However, if one takes the metaphor literally, the implied nature of the standing wave within the air column is not in accord with the truth and, hence, the positions of the nodes and antinodes are incorrectly predicted. The reader will find diagrams of the actual standing wave patterns in my recent article in *Scientific American*.[5] On several occasions I have had dealings with craftsmen and musicians who have been led astray by the metaphor (which unfortunately is to be found in many standard books). The metaphor implies that trumpet pitches may be sharpened by making enlargements at either end of the air column, an implication that has led to the considerable bewilderment of more than one instrument maker who finds his expectations reversed when he makes changes of the mouthpiece or backbore.

Conclusion

We can summarize our present understanding of the acoustical aspects of the trumpet by saying that whatever virtues an instrument may have depend ultimately on three things. First of all, it is necessary to adjust the air column shape to locate input impedance peaks at frequencies that cooperate well in forming the various regimes of oscillation. Secondly, the relative heights of the resonance peaks should be suitably arranged for ease of playing. It is here that the role of the mouthpiece and backbore becomes particularly subtle. Finally, the air column has to be adjusted so that the initially returned echoes from the bell to the mouthpiece come back in good order, to ensure a stable attack.

Looking back over the history of the development of the trumpet, one can see a curious intertwining of the activities of musicians, of instrument makers, and of scientists concerned with musical acoustics. Each has his prime purposes and his particular skills; each ideally should have some appreciation and knowledge of the activities of the others, not only in the direct furtherance of his own profession but also for the stimulation of his own broader thinking. We have repeatedly seen how the significant advances in any one of these three fields have in general come from the labors of individuals who could combine a master's skill in one area, solid competence in another, and at least a strong interest in the third. We must be grateful for the existence of such people, sensitive to hints that come to them from many directions, who have the knowledge and temperament to attempt a synthesis that can further their art.

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Notes by Virginia Benade Belveal. This piece was written at the request of Edward H. Tarr, as a projected book on the history of the trumpet to be published by Batsford, London. For several reasons the planned book did not materialize, though Art Benade sent his part to Tarr on June 8, 1973. W. T. Cardwell was to have co-authored this chapter, though the two parts were intended to be separate and identifiable. Some time in the '80s Benade added more headings and removed references to Cardwell as co-author.

Benade's two books, *Horns, Strings and Harmony* (1960) and *Fundamentals of Musical Acoustics* (1976) are available from Dover Publications as reprints that incorporate corrections added by AHB to his personal copies of the books.

I want to acknowledge the help of Joël Eymard. In the summer of 2002 he requested my permission to put a French translation of this unpublished book chapter onto his website:

http://joeleymard.free.fr/

This does two things. It makes the work available to those who read French more readily than English and inspired me to look at the original as it had been sent to Edward Tarr. It also allowed me to correct several errors in the typing (which I had done), including a mathematical mistake (found by Joël) that had been overlooked by my late husband. At this time we also added figure captions. (There is another Benade article available in French at the same website.)

Acoustic Trumpet Music. Mix with Mood Abstract 44 Aggressive 18 Airy 21 Alien 2 Angelic 1 Angry 3 Animated 166 Anxious 7 Beautiful 139 Bittersweet 3 Bouncy 105 Brave 22Â Mix with Style Acid Jazz 28 Acoustic 19 Acoustic Blues 1 Alternative Rock 2 Ambient 6 Avant Garde 10 Blues 3 Bossa nova 5 Breakbeat 1 Celtic 2 Chill out 2 Chillhop 8 Chillstep 1 Classical 17 Classical Piano 2 Country 6 Country Rock 3 Dance 40 Disco 8 Easy Listening 40 Electric Blues 2 Electronic 9 Ethnic 5 Experimental 5 Folk 20. Get Trumpet Sounds from Soundsnap, the Leading Sound Library for Unlimited SFX Downloads.Â Update Required To play the media you will need to either update your browser to a recent version or update your Flash plugin. trumpet sounds (205). Most recent Oldest Shortest duration Longest duration. Any Length < 2 sec 2 sec - 5 sec 5 sec - 20 sec 20 sec - 1 min > 1 min.