

Modelling Nonlinear Economic Time Series

by

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The application of nonlinear time series models in economic and.Â Econometrica, American Economic Review, International Economic Review, Journal of the American Statistical Association, Studies in Nonlinear Dynamics and Econometrics, Economic Journal, among others Professor Ahsley has two books with Kluwer Academic Publishers and Wiley, and four book chapters He has over 30 academic articles in outlets such as the Journal of Business and Economic Statistics, International Journal of Forecasting, Macroeconomic Dynamics, Journal of. In multivariate time-series models, X_t includes multiple time-series that can usefully contribute to forecasting y_{t+1} . The choice of these series is typically guided by both empirical experience and by economic theory, for example, the theory of the term structure of interest rates suggests that the spread between long and short term interest rates might be a useful predictor of future inflation. The multivariate extension of the univariate autoregression is the vector autoregression (VAR), in which a vector of time-series variables, Y_{t+1} , is represented as a linear function of Y_t , $\hat{\epsilon}_t$, \hat{Y}_t^{p+1} ,